PORTFOLIO PERFORMANCE: AN ACTIVE APPROACH TO WEIGHTING ASSETS IN THE PORTFOLIO VERSUS NAIVE DIVERSIFICATION

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Abstract

Modern portfolio theory represents the approach to constructing an optimal portfolio based on the weighting of individual assets in the portfolio. We can call this approach as an active, because the investor assigns higher weights to the assets that are able to achieve higher performance on the basis of past performance. In addition to the active investment approach, we also know the passive investment approach, through which the investor seeks to evenly distribute capital among other assets (naive diversification). The aim of this paper is to evaluate the performance of the active approach to weighting in the portfolio versus naive diversification. For the purpose of this work, we divide the data sample into two parts. The first part will serve to determine weights in the portfolio (training set) and in the second part we will test the performance of this portfolio versus the approach of naive diversification (testing set).

Keywords*: Markowitz, weight of assets in investment portfolio, benchmarking, active diversification, naive (passive) diversification*

JEL Codes*: G11, G17*

1. Introduction

It is known, that all investors in capital market try to find an investment strategy under which they can achieve higher expected return from invested capital. Investments with higher expected returns always come with more risk and safer investments generally pay less. If investor holds only one type of the asset, or invests in one specific sector, he is exposed to the unsystematic risk. To reduce this risk, investors diversify their portfolio by buying a variety of asset classes. Diversification is an investment strategy to manage risk and the easiest way to achieve diversification is "naive diversification", where investor spreads his capital evenly between assets in the portfolio. An active approach, known as a modern portfolio theory, which is one of the other ways to risk diversification, allows the investor to assign weights to assets in portfolio according to predefined criteria. These criteria can be set to prefer some assets in portfolio, and therefore the weights may be different. The naive diversification expresses "passive" approach to manage investor´s portfolio, because it is the easiest way to capital redistribution. More sophisticated is an “active” approach to diversification, because it includes analysis of performance measurements and according to the results it changes weights of assets in the portfolio.

2. Diversification

According to Mangram (2013) the term “diversification” refers the relationship between correlation and portfolio risk. The author explains, diversification is a cornerstone on Markowitz portfolio selection theory and modern portfolio theory is a risk reduction concept that involves the allocation of investments among various financial instruments, industries and other investment categories. In more simplistic terms, it relates to the well-known adage “don’t put all your eggs in one basket.” If the basket is dropped, all eggs are broken; if placed in more than one basket, the risk that all eggs will be broken is dramatically reduced (Fabozzi, Gupta and Markowitz, 2002).

Diversification can be achieved by the investing in different type of assets classes (e.g. bonds, real estate, etc.) and commodities such as gold or oil. The main objective of diversification is to maximize expected return and minimize risk by investment in different type of assets that would each react differently to the same events. For instance, negative news related to the European debt crisis generally causes the stock market to move significantly lower. At the same time, the same news has had a general positive impact on the price of certain commodities such as gold (Mangram, 2013). It is important, that the portfolio diversification strategies include not only the different type of assets in one specific sector, but also outside of that sector, but that they should also include different asset classes, e.g. bonds and commodities (Davidow and Peterson, 2014). Diversification Effect refers to the relationship between correlations and portfolios (Gibson, 2013).

When the correlation between assets is imperfect (positive, negative), the result is the diversification effect. It is an important and effective risk reduction strategy since risk reduction can be achieved without compromising returns (Hight, 2010).

According to Blumenthal (2014) diversification is based on diversity. If the investor´s portfolio contains instruments, which behave similarly in time, then the portfolio generally has no different characteristics compared to the individual assets in the portfolio. On the contrary, if the individual assets in portfolio behave differently in time, then the portfolio result (the risk/yield profile) may be better than the individual instruments. The ideal diversification method is to find the assets that have the lowest correlation. This study was focused on analyzing the developments of stock market index S&P 500 and bond index The Barclays Aggregate Bond Index over the years 1976-2015. The stock market index reached higher yield than bond index, but at the higher risk (measured by the standard deviation). Correlation between these indices was at low level (-0.07) over the period, which, according to the author, is optimal for diversification. The portfolio consisting of 60% of stock market index and 40% of bond index achieved better risk/yield profile compared to individual assets. Currently, the investor has several options to looking for low correlated assets such as commodity and real estate investments, hedge funds and other investment strategies.

Markowitz's portfolio theory diversifies the investor's capital among the selected assets. This theory assigns different weights to assets (based on optimization criteria), which may have a higher potential in the future, to achieve a better risk/yield profile for the investor.

2.1 Markowitz portfolio theory

Markowitz (1952) published his paper of portfolio selection, which is divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and end with the choice of portfolio. Markowitz´s portfolio theory deals with this second stage. To create an optimal portfolio, the author takes the expected return and risk into account.

In this paper we will test Markowitz´s theory, and the simple diversification between the eight selected assets will be considered as a benchmark. The Markowitz´s model is based on the following assumptions (Markowitz, 1952):

- *Rational behavior of investors:* investors focus their efforts on minimizing risk and maximizing the expected return. This assumption is inconsistent with "the herd effect", because investors are looking for special opportunities, which results in short-term fluctuations in the asset's price (Wilford, 2012).
- *Investors are willing to accept higher risk only if they are offset with higher expected returns* - the assumption that investors are willing to accept higher risks, if they are offset by higher expected returns is often inconsistent with the investor's behavior. Investment strategies often require investors to invest in risky investments (e.g. derivates or futures) in order to reduce total risk without any apparent increase in expected return (Wilford, 2012). In addition, investors perceive risk differently (their utility functions are different). It means that investors do not exceed a certain risk, threshold even if they could get a higher return on investment.
- *Investors have the same information* – this is the assumption, that all relevant information is given to investors in time, and this information is then reflected in their investment decisions. In fact, financial markets include an information asymmetry (it is a situation when one party to a transaction has more or superior information then the other party), trading on confidential information and investors who are simply better informed than others (Degutis and Novickyté, 2014).
- *Investors can borrow or lend unlimited amount of capital at risk free rate* – in fact, each investor has a credit limit.
- *Taxes and transaction costs are negligible* - from a practical point of view, investments are burdened by both aspects. Taxes and transaction costs can influence the optimal portfolio selection (Mangram, 2013).
- *Markets are efficient* - Markowitz´s theoretically papers are based on the assumption, that the markets are perfectly efficient (Markowitz, 1952). Financial markets prove their inefficiency through price jumps and information asymmetry (Degutis and Novickyté, 2014).
- *It is possible to select assets in the portfolio, whose individual performance is independent of other assets in the portfolio* – with a high level of dependency between assets, we can exclude one on these assets from the portfolio. In this case, however, there is an increase in unsystematic risk. During a period of high uncertainty (financial crises), the correlation between assets increases, and the concept of diversification then becomes limited (Alexeev and Tapon, 2012).

The investor faces a risk in constructing the portfolio. The risk can be divided to unsystematic and systematic. According to Sivák, Lauko, Gertler and Kováč (2005), this division is based on the scope of risk, respectively, whether the risks affect to all financial market instruments or just one particular. Systematic risk (market risk) affects all assets in portfolio and its level (strength) can be measured by correlation coefficient between the stocks of company and market index. This risk cannot be completely eliminated. The main sources of systematic risk are:

- *Changing interest rates* - bond prices are declining with the rise in market interest rates, and vice versa.

- *Inflation* - rising inflation causes a fall in return on investments.
- *Market risk* - the value of the investment may fluctuate due to fluctuations in world financial markets.

Mangram (2013) identifies this risk as macroeconomic (i.e. resulting from the macroeconomic situation). Apart from the above, a source of systematic risk can be, for example, changes in GDP, unemployment rate and exchange rate.

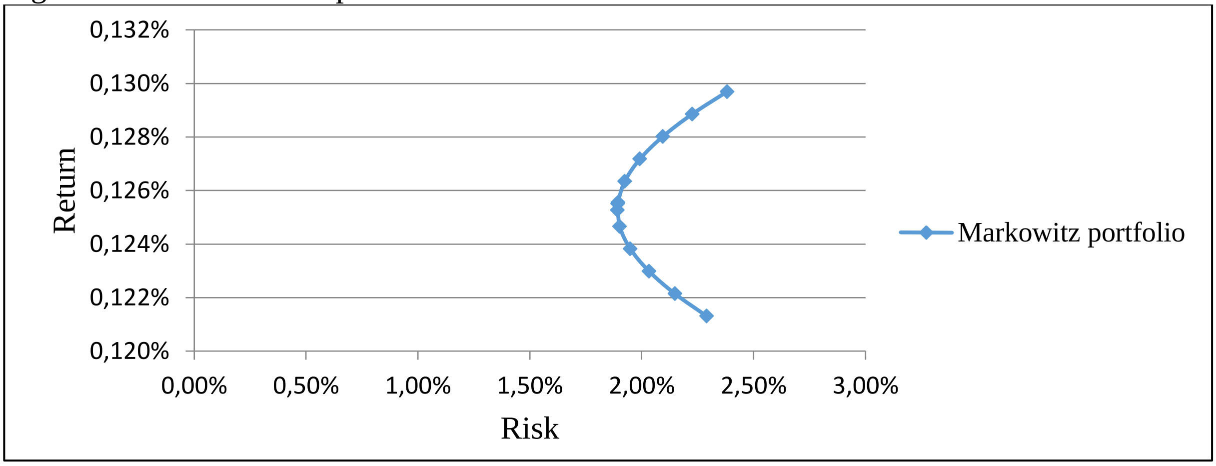
The second type of risk is unsystematic risk, which is also referred to as a unique risk and is the result of an impact on a specific company or sector. This type of risk is influenced by the factors as mismanagement of the company. Unsystematic risk can be partially eliminated by diversification of the portfolio (by increasing the number of assets in the portfolio) (Frantz and Payne, 2009). Based on these facts, then:

Total risk = systematic risk + unsystematic risk

It is important to note that a highly diversified portfolio can improve returns and significantly reduce unsystematic risk. In practice, it is very unlikely that any diversification in portfolio can effectively eliminate all risks – because, there are too many variables. In addition, no amount of diversification can remove or reduce the systematic risk that affects all companies and markets at the same time.

An effective frontier also known as “Markowitz Efficient Frontier”, is a key concept of Markowitz's theory of the portfolio. This is the best combination of assets (of those that create the maximum expected return for a given level of risk) within the investment portfolio (Dwyer, 2014). The relationship between the expected return and risk of portfolio is usually depicted by the graph comparing the risk with the expected return. Portfolios on “effective frontier” in Figure 1 represent the best possible combination of risk and expected return.

Figure 1: Mean-Variance portfolio



Source: the authors.

Portfolio selection involves investor´s preferences for risk. Investor that is highly averse to risk chooses the portfolio that is at the top of the parabola because it represents the lowest level of risk. On the contrary, if investor is willing to accept higher risk, his portfolio will move after the parabola from its peak (top). The portfolios at the bottom of the parabola are not efficiency, because for these portfolios, it is possible to find portfolios with the same risk profile but with a higher expected return. The portfolios at the top of the parabola are located at an effective frontier.

2.2 Naive diversification and advanced models of diversification

The principle of diversification consists in redistributing the investor´s capital into the various instruments. The basic capital distribution approach is naive diversification – the principle of $1/N$ - what means, that the investor evenly distributes his capital into the N financial instruments. DeMiguel, Garlappi and Uppal (2009) compared 14 different diversification models with $1/N$ principle. They used 8 different stock datasets to run 14 different diversification models, which they called as optimal portfolios. All these models were based on the different percentage capital distributions between financial instruments with regular portfolio rebalancing. The result of the study was the fact, that the more sophisticated active approach to diversification did not confirm higher Sharpe ratio and return on investment, relative to naive diversification. They also pointed to the fact, that if the portfolio is constructed of 25 assets, it would take 3000 months to overcome the naive diversification by the more sophisticated active approach. The authors explain these results by estimation error.

Hwang, Xu and In (2017) also tested the performance of naive diversification compared to the optimal portfolio. Like DeMiguel, Garlappi and Uppal (2009), they demonstrated that the naive diversification overcame by its performance any of the optimal portfolios. They argue that the portfolio created on the base of naive diversification is characterized by increased exposure in the left tail. It means that such a portfolio tends to have not only a negative coefficient of skewness, but also an increased positive coefficient of kurtosis compared to normal distribution. Naive diversification makes portfolio returns more concave, so naive diversification tends to overcome the Markowitz Mean-Variance portfolio. The last factor that results in an increase in performance is associated with the number of assets held in the portfolio. The higher the number of assets in the portfolio makes more concave portfolio returns.

According to Tu and Zhou (2011) Markowitz´s modern theory of portfolio and all its extensions lag behind the $1/N$ principle, what is in line with the results of previous authors. The authors understood the concept of total portfolio diversification as a combination of $1/N$ principle with four other models. Ultimately, the authors achieved better results of the risk-yield profile compared to the $1/N$ principle.

3. Data and methodology

In this paper we focus on Markowitz optimal portfolios with eight selected assets. The expected return on the portfolio is given by:

$$\overline{r}_p = \sum_{i=1}^{n} \overline{r}_i w_i ,$$

where $\overline{r}_i$ is mean value of the asset return in the portfolio and w_i is the weight of the asset in the portfolio.

Portfolio return is the weighted arithmetic mean of the individual assets returns in the portfolio. Portfolio risk is measured by the variance, where covariance matrix represents link between assets. Portfolio spread is given by:

$$\sigma^2{}_p = \sum_{i,j=1}^{n} w_i w_j \sigma_{ij} ,$$

where w_i is the weight of the asset in the portfolio and σ_{ij} is covariance between assets in the portfolio.

3.1 Naive diversification and optimization criteria of Markowitz portfolios

Naive diversification is a strategy where the weights of assets in the portfolio are same for each asset ($w_i = 1/N$). This strategy ignores information from data and does not include any performance parameter, estimates and does not consider short selling. This approach can also be called passive, because the investor does not attach greater weight to "better" assets.

We classify the composition of optimal portfolios as an active approach, because the higher weights are assigned to those assets that are more capable of generating higher performance parameters than others in the optimization task and we do not allow short selling in construction of optimal portfolios. Table 1 lists various types of optimization tasks based on the Markowitz portfolio theory. The first “MinRisk” strategy is a strategy that minimizes standard deviation of the portfolio. The second strategy generates the portfolio which is located at the Markowitz effective frontier (the upper part of the parabola). In this strategy we require expected return equals to 0.1%. The "MaxRF" strategy takes into account both investment components: return and risk and is focused on maximization recovery factor indicator ($RF = \bar{r}_p / \sigma_p$). The last two strategies are based on the "Safety First" rule. Based on this rule, we concentrate on eliminating bad results (losses from investing). For the Kataoka criterion, we require that probability of daily expected return of the portfolio falls below a certain threshold is not greater than 0.05 (Inverse of the standard normal cumulative distribution, with a probability of 0.05, u_α = -1.644). In constructing the optimal portfolio according to the Telser´s criterion, we require, that the probability that the daily loss is more than 2% is not greater than 0.05. These criteria are represented by left-side confidence level of return ($r_L = \bar{r}_p + u_\alpha \sigma_p$).

Table 1: Optimization criteria of Markowitz portfolios

Description of strategy	Code of strategy	Optimization criteria
Portfolio with the lowest risk	MinRisk	Optimization task: $\min \sigma^2$ where: $\sum_{i=1}^{n} w_i = 1$ $w_i \geq 0$
Risk of a given return $(E_p = 0.1\%)$	EfektFront	Optimization task: $\min \sigma^2$ $\bar{r}_p = 0.1\%$ where: $\sum_{i=1}^{n} w_i = 1$ $w_i \geq 0$
Maximizing the Recovery Factor Indicator	MaxRF	Optimization task: $\max \frac{\bar{r}_p}{\sigma_p}$ where: $\sum_{i=1}^{n} w_i = 1$ $w_i \geq 0$

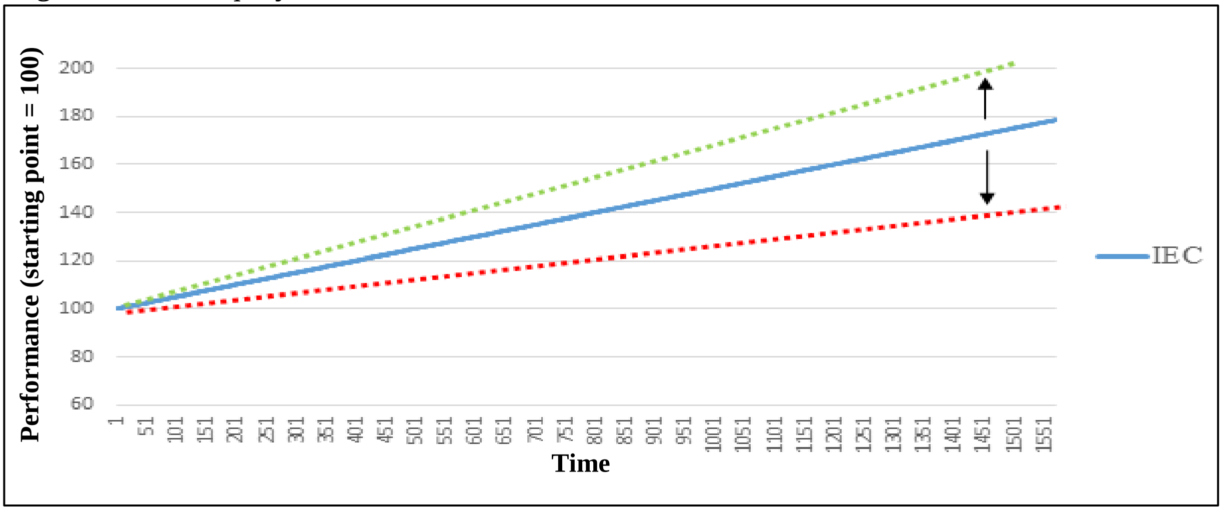
Portfolio using the Telser´s criterion	TeslerK	Optimization task: $\max \bar{r}_p$ $\sum_{i=1}^{n} w_i = 1$ where: $\bar{r}_p + u_\alpha \sigma_p \geq r_L$ $w_i \geq 0$
Portfolio using the Kataoka criterion	KataokaK	Optimization task: $\max r_L$ where: $\sum_{i=1}^{n} w_i = 1$ $w_i \geq 0$

Source: the authors.

3.2 Ideal Equity Curve

If the portfolio is constructed only from loss equity curves (e.g. assets, strategies), the result of naive diversification will also be just a loss. In order to avoid loss investment instruments ("why to invest in loss ones?"), we created a hypothetical equity curve that has a purely linear growing trend as is depicted in the Figure 2. It is represented by an Ideal Equity Curve (IEC) that is hypothetical curve (hypothetical benchmark) and our own idea. From the point of view of investor, IEC represents at least "wanted" equity curve performance in the future. Investor can determine the "wanted" angle of IEC by himself according to his preference. Surely, there is no way to invest in IEC and it serve only for the needs of the decision algorithm.

Figure 2: Ideal Equity Curve



Source: the authors.

The algorithm using the IEC is based on its comparison with the performance of the individual investment instruments from the portfolio over time. If the performances of the individual investment instruments over time are above IEC, the algorithm allows investing in the relevant instrument, and vice versa, if the performances of the individual instruments are under IEC, the algorithm stop investing to it (process of creating IEC portfolio). By setting the IEC slope to a higher (green curve in the Figure 2) and to the lower (the red curve in the

Figure 2), it is possible to regulate requirements placed on the performance of the individual investment instruments by investor. The higher the requirements on performance are, than more often investments instruments are "shut down" (i.e. they will be not in the IEC portfolio). This algorithm protects investor from investing in potentially troubled instruments, so investor´s capital is invested only in “good” or “wanted” ones.

However, in some situation there is better naive diversification (passive approach) from the point of view of the Total return. For example if all assets in the portfolio are in growth trend. These findings have led us to consider connecting and using both approaches. First –active, which is able to “filter” risk of the portfolios (using IEC concept). Second - passive, which, in the case of “good” composition of the portfolio has better performance.

The fundamental idea of the Final strategy is to compare both approaches with the fact that it “favors” and invests in the concept that is better in terms of profitability over time.

If the return of the naive diversification over time is under the portfolio created by using IEC, the Final strategy invests in portfolio created by using IEC concept. On the other hand, if the return of the naive diversification is above the portfolio created by IEC, the Final strategy invests in passive approach (naive diversification). It means that if the naive diversification outperforms IEC portfolio, it is unnecessary to invest only in “good” ones and investor should choose naive diversification. On the other site, if the IEC portfolio is better than naive diversification, investor should choose IEC portfolio. The Final strategy is so the choice between passive approach (naive diversification) and active approach (IEC portfolio).

In this way, the investor would be able to protect himself against the choice of potentially only loss-making investment instruments, and at the same time he would not miss the potential return from the passive approach.

3.3 Data

Table 2 lists the companies selected for analysis. All analyzed companies are traded on the New York Stock Exchange or NASDAQ. Portfolios are created by using close prices of selected companies. The monitored companies operate in six different sectors of the economy. By this selection, we have achieved a relative sectoral diversification. The observation time is from January 4th, 2010 to March 31th, 2016. We chose this time period on purpose, because it shows the opposite performance of the training sample portfolios versus the test sample portfolios.

Table 2: Name, ticker and sector of analyzed companies

Full name of company	Ticker	Sector
Oil-Dri Corporation of America	ODC	Basic Materials
Fresenius Medical Care AG & Co. KGaA	FMS	Healthcare
TE Connectivity Ltd.	TEL	Technology
JetBlue Airways Corporation	JBLU	Services
The Hanover Insurance Group, Inc.	THG	Finance
ePlus inc.	PLUS	Technology
Dillard's, Inc.	DDS	Services
Valero Energy Corporation	VLO	Basic Materials

Source: the authors.

We monitor the level of total return and risk (measured by standard deviation) and the maximum drawdown over time. A drawdown is the peak-to-trough decline during a specific recorded period of an investment. Maximum drawdown means the highest possible loss that the investor had to face over time. We divide the data base into two groups. The first group consists of the data on which we assign the weight to the individual assets in the portfolio (the training sample - from January 4th, 2010 to December 31th, 2014). On the test sample (2nd data set - from January 2nd, 2015 to March 31th, 2016) we will test the ability of optimal portfolios to generate profits. The basic benchmark is the strategy of naive diversification (passive approach). We built and evaluated portfolios in MS EXCEL.

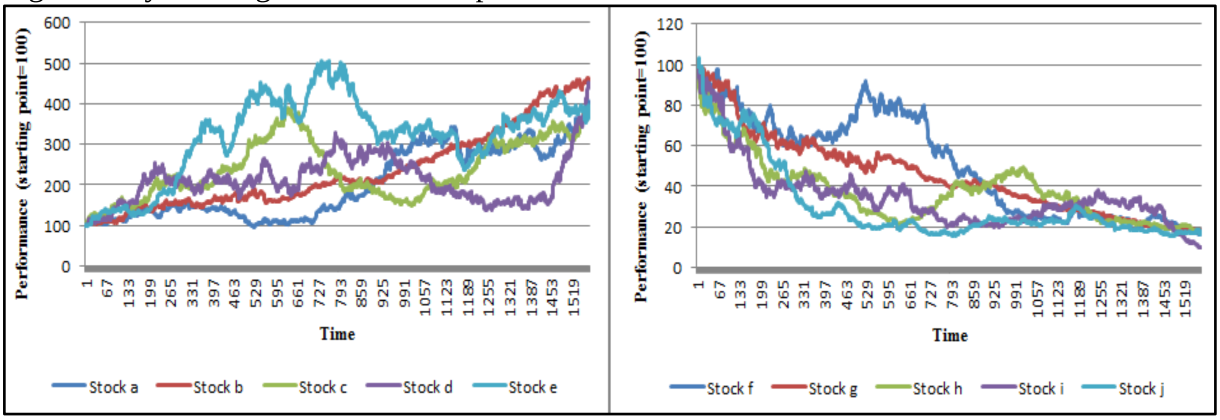
3.4 Simulated data

For the purposes of closer research, we have also used own generator that artificially simulate the development of individual equity curves (Synthetic generated stocks) in 1570 business days. This data allowed us to test IEC concept also in situations where the portfolio was not created only by using stocks characterized by a growth trend.

The simulator (Synthetic generated stocks) is based on the daily changes in ODC return. This stock performance was chosen randomly from other previous analyzed stocks. The principle of the generator lies in the random distribution of daily deviations calculated via MS EXCEL from the real daily ODC data for the entire monitored period.

Synthetic generated stocks (SGS) were divided into two groups, and each group representing a different set of SGS characterized by another trend. The first group is characterized by the fact that during the period under review the SGS was characterized by a growth trend (Stock A, B, C, D and E). The second set consists of those SGS that have a decrease trend in period under review (Stock F, G, H, I and J). In Figure 3 the performances of both groups are depicted. Correlation matrixes in Table 3 were made by returns obtained from SGS data.

Figure 3: Synthetic generated stocks performance



Source: the authors.

Table 3: Correlation between growth synthetic stocks and decrease synthetic stocks

	Stock a	Stock b	Stock c	Stock d	Stock e
Growth synthetic stocks					
Stock a	1	0.006	0.000	0.034	-0.019
Stock b	0.006	1	0.020	0.013	-0.046
Stock c	0.000	0.020	1	0.006	0.028
Stock d	0.034	0.013	0.006	1	-0.033
Stock e	-0.019	-0.046	0.028	-0.033	1
Decrease synthetic stocks					
	Stock f	Stock g	Stock h	Stock i	Stock j

Stock f	1	0.006	0.000	0.034	-0.019
Stock g	0.006	1	0.020	0.013	-0.046
Stock h	0.000	0.020	1	0.006	0.028
Stock i	0.034	0.013	0.006	1	-0.033
Stock j	-0.019	-0.046	0.028	-0.033	1

Source: the authors.

4. Results and Discussion

4.1 Weights in Markowitz portfolio

Table 4 shows the weights of individual companies in the portfolios. In some portfolios, certain stock titles have been excluded from portfolios, which increase in non-systematic risk. The highest weighted concentration case is observed for the EfektFront portfolio, which is aimed at minimizing risk at the expected return level ($\overline{r}_p = 0.1\%$). This asset concentration is too high and the diversification effect is weaker. The portfolio focused on risk minimization (MinRisk) has also a high concentration within two assets (THG and FMS) that have reached the lowest level of risk measured by the standard deviation. Weights in KataokaK and MinRisk portfolio are similar. It means that their performance and risk will be similar, too.

Table 4: Weights in created portfolios

Strategy Code	**ODC**	**FMS**	**TEL**	**JBLU**	**THG**	**PLUS**	**DDS**	**VLO**
MinRisk	9.81%	36.14%	6.98%	0.00%	38.17%	4.51%	4.39%	0.00%
EfektFront	0.00%	0.00%	0.00%	5.47%	0.00%	39.62%	31.74%	23.17%
MaxRF	5.57%	0.60%	3.85%	5.59%	22.80%	25.84%	20.83%	14.92%
KataokaK	9.73%	35.17%	7.09%	0.00%	37.88%	5.18%	4.94%	0.00%
TeslerK	6.18%	7.59%	4.47%	3.74%	27.70%	21.43%	17.42%	11.47%
NaiveDiv	12.50%	12.50%	12.50%	12.50%	12.50%	12.50%	12.50%	12.50%

Source: the authors.

4.2 Performance of portfolios

In this part of the paper we apply selected methods of diversification which were presented at the Table 1. In these portfolios, we mainly monitor the ability of optimal portfolios to generate higher performance than naive diversification (benchmark for these portfolios).

Table 5 lists the results of a training data sample analysis. In the case of eight selected titles, the increased performance of naive diversification over all other strategies did not appear over time. The performance of naive diversification is higher only when compared to the MinRisk and KataokaK strategies. These strategies are aimed at limiting risk, which is also visible at the value of maximum drawdown. Investor's attitude to risk therefore influences his choice of portfolio. If the investor accepts higher risk of investment over time (e.g. 10 bp), the potential return is approximately 4 times higher (EfektFront strategy). The EfektFront portfolio achieves the highest levels of performance but also risk. This relationship thus confirms Markowitz portfolio theory, where the higher return of the strategy means higher risky portfolio.

Table 5: Performance of training set portfolios

	MinRisk	EfektFront	MaxRF	KataokaK	TeslerK	NaiveDiv
Return	84.02%	308.66%	219.0%	87.07%	185.31%	166.73%
Average daily return	0.05%	0.13%	0.1%	0.06%	0.09%	0.09%
Standard deviation	1.07%	1.67%	1.39%	1.07%	1.29%	1.28%
Max. drawdown	-20.10%	-30.78%	-25.60%	-19.99%	-23.46%	-26.04%

Source: the authors.

Table 6 lists the performance of the testing set portfolios. The portfolios have been recalculated by the same weights, determined on the data from the training set. The strategies that were the most successful in training data (reaching the highest return) are weak in the performance of the test sample. An investor, who select for a top-rated portfolio (EfektFront), would end up with a slight loss.

EfektFront portfolio, there is up to 94% of the weight is spread between three assets and, in the case of MaxRF, it is approximately 70%. These two portfolios are aggressive because the aim of their optimization tasks was to maximize performance, respectively determined return and weights in these portfolios were concentrated on more risky assets. The fall in the prices of some assets cause the low efficiency of the EfektFront strategy due to the high concentration on three titles (PLUS, DDS, VLO).

Table 6: Performance of testing set portfolios

	MinRisk	EfektFront	MaxRF	KataokaK	TeslerK	NaiveDiv
Return	16.82%	-0.42%	7.10%	16.41%	9.34%	9.60%
Average daily return	0.06%	0.01%	0.03%	0.05%	0.04%	0.04%
Standard deviation	1.05%	1.54%	1.25%	1.05%	1.16%	1.16%
Max. drawdown	-10.91%	-30.78%	-20.11%	-10.89%	-17.52%	-15.40%

Source: the authors.

We think that there is a chance to be better than naive diversification in short time period. Optimal portfolios are always exposed to systematic risk, i.e. the risk arising from the macroeconomic situation. Several portfolios achieved a higher concentration of weights in stocks of individual company. The lower degree of diversification increases the presence of a unsystematic risk that is more difficult to measure. As a result, the downturn in asset prices has led to more drastic changes in the overall portfolio. Fluctuations in the financial markets are a normal part of the investment and each trader must bear them. There was always the risk from equity investments that can be eliminated to a certain extent but never eliminated altogether.

Predictive ability of the training sample did not appear. Investors who would choose the most profitable portfolio would end up with the lowest profits. In the case of the test sample, the increased efficiency of the naive diversification did not occur. Our analysis does not imply a preferential portfolio that has overcome its characteristics over others. This situation can be caused because our estimation of weights was made by the historical data. The results also point that past performance does not guarantee future results.

4.3 Ideal equity curve as tool of active management

In this part we test the functionality of IEC algorithm:

- firstly, on eight stock data that were also used in Markowitz's portfolios,
- secondly, on simulated data (Synthetic generated stocks).

Firstly, we set IEC as a constant "zero" line. In this case we required that investment performance cannot fall behind this line. All of 8 analyzed stocks have growth trend in period under review, so there were only a few situations where IEC algorithm stopped investing in some stock. As a result, in this situation Total Return of naive diversification and Final strategy were very similar, but naive diversification won.

Then we run one hundred simulations with always different angle of IEC. Results of this testing brings Table 7. We found out that in case of eight only growth equities, there was no situation where Final strategy outperformed naive diversification.

Table 7: Performance compilation of Naive diversification and Final strategy in 100 simulations

	Naive diversification	**Final strategy**
Total average return	294.79%	260.85%
Number of wins	100	0
Maximum difference (coefficient for IEC=0.2988)	294.79%	81.66%
Minimum difference (coefficient for IEC=0.0263)	294.79%	293.99%
Total Return (coefficient for IEC =0)	294.79%	293.99%

Source: the authors.

Previous results show that for eight stocks with a growth trend there was no possibility of outperforming naive diversification (naive diversification won 100 times). The total average return of naive diversification from one hundred simulations was higher than in case of Final strategy (294.79% vs 260.85%). From all simulations the worst result for Final strategy was when coefficient for IEC was set on 0.2988 (profit only 81.66%). The best result achieved Final strategy when coefficient for IEC was set on "zero" or 0.0263 (profit 293.99%). These results show that higher angle of IEC caused poor performance of Final strategy (we explain this by more "false signals").

In connection with previous findings, we tested Final strategy algorithm also on simulated data. This data allowed us to test IEC also in situations where the portfolio was not created only by using growth stocks. For these purposes we created five growth equity curves and five equity curves with decrease trend (see part 3.4 Simulated data).

Final portfolio was made from SGS by five randomly selected equity curves. We developed own random selecting formula to ensure:

a) there is at least 1 equity curve with decrease trend in final portfolio,
b) that randomly constructed final portfolios will be always different (will be always composed from 5 different SGS).

The total number of analyzed final portfolios was 102. First final portfolio was made from all 5 growth SGS and second from all 5 SGS with decrease trend. Other 100 final portfolios were made via our random selecting formula. All 102 combinations were tested with coefficient for IEC set on 0 and also on 0.0297. The second coefficient was counted as "wanted" MSCI World Index daily return. Table 8 shows the results of our analysis.

Table 8: Results from 102 combinations

Condition	Coefficient for IEC=0.0297	Coefficient for IEC=0
NaiveDiv (+) and Final Strategy (+)	65	65
NaiveDiv (-) and Final Strategy (+)	36	36
NaiveDiv (+) and Final Strategy (-)	0	0
NaiveDiv (-) and Final Strategy (-)	1	1
Total Return (102) NaiveDiv	35.37%	35.37%
Total Return (102) Final Strategy	332.62%	342.68%

Source: the authors.

Performance of naive diversification is always constructed as an average of individual SGS performance. If there is the SGS in the portfolio with decrease trend, it negative affects investment results from naive diversification. But in this case thanks to IEC - Final strategy algorithm stopped investing to decrease trend SGS, so Final strategy can outperforms naive diversification.

From 102 combinations there were 65 situations where both of strategy (naive diversification and Final strategy) had positive final return. There were 36 combinations where the naive diversification returns were negative, but Final Strategy return was positive. There were 0 situations, when naive diversification returns were positive and Final strategy return was negative.

From selected combinations there was only 1 combination, where performance of both strategies had negative final return. This combination was made from all five decreased SGS. However both strategies had negative final return, we have to point at the fact that performance of Final strategy was only slightly negative (loss – 3.28%) and the performance of naive diversification was strongly negative (loss – 81.62%). In this situation Final strategy algorithm was able to protect investment capital and naive diversification was not.

The combination when portfolio was created by all five growth SGS is the only situation where naive diversification outperformed Final strategy. It is in coincidence with our first tests with eight stock data that were also used in Markowitz's portfolios (all 8 were growth). In case of 5 growth SGS, the difference between strategies was 36.43% (for IEC coefficient= 0.0297) and 5.28% (for IEC coefficient= 0), in favor for naive diversification.

Final strategy algorithm seems to be effective if the portfolio is created by at least with one downtrend equity. From the point of view of Total return, Final Strategy got from 102 combinations better results than naive diversification (approximately 10 times higher return).

5. Conclusion

In this paper the five optimization tasks of active approach to weighting in portfolio is presented by using Markowitz portfolio theory. We found out, that the training sample of three portfolios – EfektFront, MaxRF and TeslerK - achieved higher performances and conversely, in the testing sample (in forwardtest), the opposite portfolios - MinRisk and KataokaK – obtained better results. We can conclude that the predictive ability of the training sample did not appear, so investors who would choose the most profitable portfolio would end up with the lowest profits, and the position against naive diversification was in longer data sat totally the opposite.

According to several authors like Hwang, Xu and In (2017), DeMiguel, Garlappi and Uppal (2009) and others, the naive diversification is the best (not only easiest) choice for the investor and any attempt to an active approach to diversification is unsuccessful in long-term.

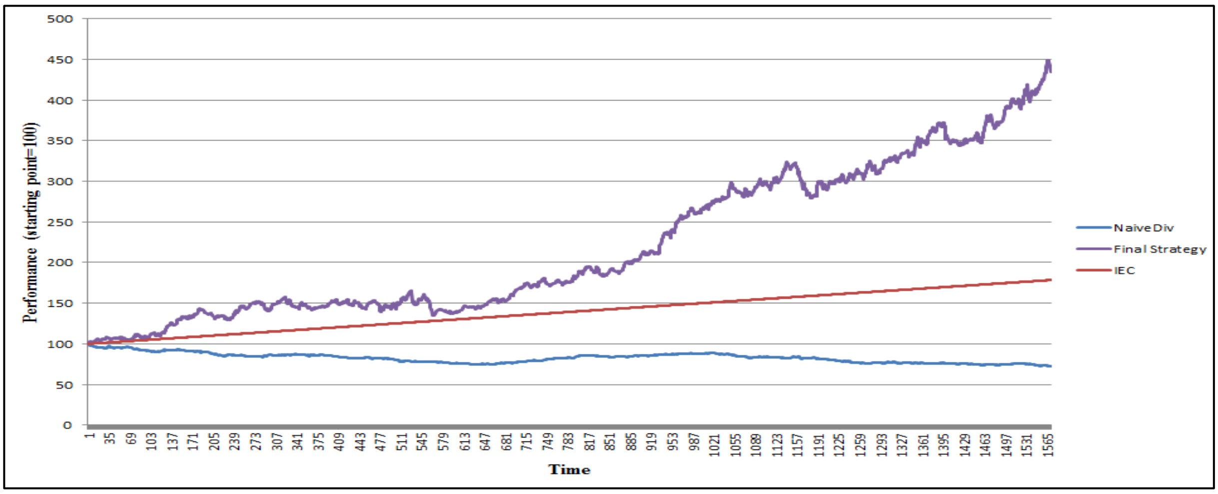
Our five optimizing tasks are in line with these findings, and moreover we confirmed the fact, that the past development is no guarantee of the same development in the future.

We do not deal with the fact if the naive diversification proves higher performance in long-term than any other active approach to diversification of the portfolio or not. However, we assume that the findings of these authors are based on the truth, since empirical data on the performance of active and passive managed funds in several studies is clearly in favor of passive. It seems that the naive diversification is the best choice for any rational investors. However, it is only theoretical considerations that naive diversification may also be a bad decision for the investor if he chooses only the assets that will decrease in the future. In that case he will not achieve profit by the diversification, but only the loss.

The mentioned problem was solving by using the synthetic benchmark – IEC. With this benchmark, the investor can not only avoid this, but also profit. The philosophy of using IEC is very simple. This benchmark serves to select assets that belong to the portfolio by performance and assets which are no longer. In simplicity, it also determines assets and their weights. We tested the functionality of the IEC concept on artificially generated ten stocks performance data that we divided into two basic groups – "growth" and "loss". By using a simple algorithm we simulated their random selection into a hypothetical portfolio. The first portfolio always consisted of testing the set of all growth, the second of the set of all drops, and the portfolio of the next 100 simulations determined by the random selecting formula to eliminate any duplicity in the portfolio composition.

As a result of testing the functionality of our IEC concept, the resulting total return of 102 simulations was much higher in the Final Strategy than in naive diversification. And, in 36 cases (35%), naive diversification in loss occurred, while the Final strategy was even in pluses. An example is given in the next figure. The algorithm randomly selected 5 artificially generated stock data - 3 drop stocks (Stock F, G and H) and 2 growth stocks (Stock A and B). Naive diversification only invested 20% in each stock. But, thanks to our active approach (Figure 4), Final strategy not only overcame naive diversification, but it has also generated a profit (as opposed to the naive diversification that was loss).

Figure 4: IEC, Final strategy and naive diversification took from Stock A, Stock B, Stock F, Stock G and Stock H



Source: the authors.

The IEC algorithm does not actively change weights in portfolio when all investments overcome the IEC. The result from this situation is only naive diversification. However, if any investment drops below the IEC, algorithm excludes these investments from the portfolio and increases the remainder weights proportionally. The basic element of the whole concept is the protection of invested capital over time. Possible profit is only a positive externality resulting from the random selection of those investments that will overcome the IEC.

Acknowledgement
The paper arose in fulfilment of the grant scheme VEGA 1/0554/16 Alternatives to replicating portfolio selection in the conditions of frequent rebalancing of the Ministry of Education, Science, Research and Sport of the Slovak Republic.

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