

## A PAYOUT PRODUCT WITH INCREASING PAYMENTS IN THE OLD-AGE PENSION SAVING SCHEME IN SLOVAKIA

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### Abstract

*The Act 43/2004 Coll. on the Old-Age Pension Saving Scheme and on amendments and supplements to certain laws offers, among others, an old-age pension product with increasing annuity payments. In our model we consider a monthly paid annuity increasing yearly in geometric progression. Just the determination of the geometric progression quotient is a crucial task, especially when we consider valuation of the future cash-flows based on risk-free bond yield curves. On the basis of actuarial formulas we point out on a mutual relationship between the risk free bond yields and the yearly increase rate of the pension payments. In our contribution we also examine the impact of the increase rate on the overall benefit of the pensioner.*

**Key words:** *old-age pension saving scheme, increasing annuity, geometric progression, increase rate, risk-free yields*

**JEL Codes:** *G22, G28, C6, K20*

### 1. Introduction

Old-age pension saving represents a capitalization based on contributions. This means that the amount of pension will depend on the contributions of the future pensioners paid to the pension funds. The old-age pension saving scheme in Slovakia is regulated by Act 43/2004 Coll. on the Old-Age Pension Saving Scheme and on amendments and supplements to certain laws (hereinafter Act), with status as of February 1, 2018. In modelling and analysis of the products of the old-age pension saving scheme we also respect the requirements and recommendations stated by the Act, Act 39/2015 Coll. on insurance and amending certain laws, the Council Directive 2004/113/EC of 13 December 2004 implementing the principle of equal treatment between men and women in the access to and supply of goods and services, the Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) and the Directive 2014/51/EU of the European Parliament and of the Council of 16 April 2014, better-known as Omnibus II Directive.

The Article 46 of the Act declares, "Offer of an old-age annuity and offer of an early retirement annuity", paragraph (1), as the second product (Product 2) - an old-age annuity or an early retirement annuity which includes raising of the pension but does not include survivors' benefits.

This paper represents our ongoing work on the products of the payout phase of the old-age pension saving scheme in Slovakia. In our contribution we use the same notations and relevant actuarial formulas how in our previous papers to form one research role.

This paper is organized as follows. In Section 2 we give basic notations, concepts, formulas related to actuarial modelling and definitions of the terms such as a fractional age assumption, the Lee-Carter model of longevity and the Svensson yield curve. In Section 3 we introduce complete modelling of the Product 2 and its analysis regarding to a progression rate. In Section 4 we give conclusions and our plans to further investigation of other products related with the pension annuities.

## 2. Preliminaries

Similarly, as we modelled Product 1 in our previous work, now, we offer an actuarial modelling of Product 2 taking into account all applicable acts and regulations. For a modelling of this product we apply mortality tables from years 1950-2014 and predictions on 2015-2063, of total population (The Human Mortality Database, 2017) and their adjustment according to the Lee-Carter model of longevity, (Hyndman, 2014). Moreover, the model includes market-consistent cash-flow valuation based on the Svensson yield curve whose parameters the European Central Bank (ECB) publishes on a daily basis. We apply the Svensson yield curve on 7 December 2017, for AAA rated bonds (European Central Bank, 2017). Moreover, we use linear interpolation between integer ages, the uniform distribution of deaths assumption.

In the following subsection we recall basic actuarial notations and definitions related to our modelling.

### 2.1 Basic probabilities

The basic building block in modelling of all life insurance products represent the relevant survival and mortality probabilities which are given as follows:

- ${}_t p_x$  - the probability that individual at age  $x$  survives at least to age  $x + t$ ,
- ${}_t q_x$  - the probability that individual at age  $x$  dies before age  $x + t$ ,
- ${}_{r|t} q_x$  - the probability that individual at age  $x$  survives  $r$  years, and then dies in the subsequent  $t$  years, that is, between ages  $x + r$  and  $x + r + t$ .

The probability  ${}_{r|t} q_x$  is also called a deferred mortality probability, because it is the probability that death occurs in some interval following a deferred period. It can be calculated by formula (Dickson *et al.*, 2009)

$${}_{r|t} q_x = {}_r p_x - {}_{r+t} p_x. \quad (1)$$

Because our model is based on monthly benefits and we have the annual probabilities of death, we also recall a fractional age assumption (Dickson *et al.* 2009) as follows.

**Definition 1** For integer  $x$ , provided the uniform distribution of deaths in every age interval  $[x, x + 1]$ , and for  $0 \leq s < 1$ , assume that

$${}_s q_x = s \times q_x. \quad (2)$$

We also recall the definition of the Lee-Carter model of longevity (Lee and Carter, 1992).

**Definition 2** *Lee-Carter model is defined by*

$$m_{x,t} = \exp(a_x + b_x \times k_t + \varepsilon_{x,t}), x \in \mathcal{X}, t \in \mathcal{T}, \quad (3)$$

where

- $m_{x,t}$  – the central mortality rate of a person at age  $x$  in year  $t$ ,
- $a_x$  – age-specific model parameter that do not depend on time,
- $b_x$  - age-specific model parameter that represents how rapidly or slowly mortality at each age varies when the general level of mortality changes,
- $k_t$  – time-varying index (independent of age), reflecting the general level of mortality,
- $\varepsilon_{x,t}$  – random noise (error term) with zero mean value and variance  $\sigma^2$  for  $x \in \mathcal{X}$  and  $t \in \mathcal{T}$ , where  $\mathcal{X}$  is the set of ages and  $\mathcal{T}$  is the set of times.

Estimating the Lee-Carter model parameters and predicting future mortality rates are performed in the R software (R Core Team, 2016), using the `demography` package, (Hyndman *et al.*, 2014).

## 2.2 Svensson yield curve

On the basis of Technical notes of the ECB we recall the formula of the Svensson yield curve (European Central Bank, 2017).

**Definition 3** *The Svensson yield curve is given by*

$$R(z) = \beta_0 + \beta_1 \times \frac{1 - \exp\left(-\frac{z}{\tau_1}\right)}{\frac{z}{\tau_1}} + \beta_2 \times \left[ \frac{1 - \exp\left(-\frac{z}{\tau_1}\right)}{\frac{z}{\tau_1}} - \exp\left(-\frac{z}{\tau_1}\right) \right] + \beta_3 \times \left[ \frac{1 - \exp\left(-\frac{z}{\tau_2}\right)}{\frac{z}{\tau_2}} - \exp\left(-\frac{z}{\tau_2}\right) \right], \quad (4)$$

where

- $R(z)$  – yield from a bond investment with continuous compounding (% p.a.),
- $z$  – term to maturity,  $z \in ]0, T_{max}]$ ,
- $T_{max}$  – maximum term to maturity,
- $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$  – parameters of the Svensson yield curve, where  $\beta_0, \tau_1$  and  $\tau_2$  must be positive.

The yield curve in Definition 3 is a parametric model which specifies a functional form for the spot interest rate and consists of four components. The parameters of this model can be interpreted as follows (Aljinović et al., 2012):  $\beta_0$  is the long-term asymptotic value (for very long maturities) of  $R(z)$ ;  $\beta_1$  is the spread between the long and short term and hence  $\beta_0 + \beta_1$  is equal to the short-term rate (the rate at zero maturity). Furthermore,  $\tau_1$  specifies the position of the first hump or U-shape;  $\beta_2$  determines the magnitude and direction of the hump. Parameter  $\beta_3$  is analogous to  $\beta_2$  and  $\tau_2$  can be interpreted as determining the magnitude and direction of the second hump or U-shape.

### 3. Payout product with geometrically increasing payments

In this section we give an actuarial model of the amount of the monthly pension annuity of Product 2. Firstly, we recall the basic notations which are as follows:

- $S$  - accumulated sum in the Old-Age Pension Saving Scheme, gross single premium;
- $P(z) = \exp\left(-\frac{R(z)}{100\%} \times z\right)$  - discounting factor;
- $x$  - retirement age;
- $j$  - geometric progression quotient as a % p.a.;
- $\omega$  - maximum age to which a person can live to see (regarding used life tables:  $\omega = 110$  years);
- $\alpha$  - initial costs as a % from yearly annuity;
- $\delta_1$  - administrative expenditures as a % from accumulated sum in a case of death during the first month;
- $\delta_2$  - administrative expenditures as a % from the expected present value of the sum of not yet paid monthly annuities in the case of the beneficiary death during the period of the first seven years of pension payment;
- $\beta$  - administrative costs as a % p.m. from technical provisions on whole life geometrically increasing annuity in advance;
- $IC$  - initial costs as an absolute amount in monetary units independent on an accumulated sum.

#### 3.1 Basic equivalence equation of Product 2

The basic equivalence equation represents the expected present values of all cash-flows related to basic monthly pension annuity  $MP_x$  which is geometrically increasing by  $j\%$  per

year. Hence we have

$$\begin{aligned}
 S &= 12 \times MP_x \times (\tilde{I}a)_x^{(12)} + A_{x:\overline{1/12}|}^1 \times S \times \left(1 - \frac{\delta_1}{100\%}\right) + A_{x:\overline{1/12}|}^1 \times S \times \frac{\delta_1}{100\%} + \\
 &+ 12 \times MP_x \times \left(IMA^{(12)}\right)_{x:\overline{7}|}^1 \times \left(1 - \frac{\delta_2}{100\%}\right) + \\
 &+ 12 \times MP_x \times \left(IMA^{(12)}\right)_{x:\overline{7}|}^1 \times \frac{\delta_2}{100\%} + \\
 &+ 12 \times MP_x \times \frac{\alpha}{100\%} + IC + 12 \times MP_x \times \frac{\beta}{100\%} \times \sum_{t=1}^{12(\omega-x)} \left. \frac{t}{12} \right| (\tilde{I}a)_x^{(12)},
 \end{aligned} \tag{5}$$

where

- on the left-hand side is an accumulated sum  $S$  which represents a single premium of this product.
- On the right-hand side the value  $12 \times MP_x \times (\tilde{I}a)_x^{(12)}$  is the expected present value of whole life monthly annuity in advance geometrically increasing by  $j\%$  per year, where

$$(\tilde{I}a)_x^{(12)} = \frac{1}{12} \times \sum_{r=0}^{\omega-x-1} \sum_{i=1}^{12} \left(1 + \frac{j}{100\%}\right)^r \times \left. \frac{12 \times r + i}{12} \right| p_x \times P\left(\frac{12 \times r + i}{12}\right) \tag{6}$$

is the expected present value of whole life benefits in advance of  $1/12$  of monetary units (m.u.), 12 times per year, geometrically increasing by  $j\%$  per year, conditional upon the life beneficiary.

- The expression  $A_{x:\overline{1/12}|}^1 \times S \times \left(1 - \frac{\delta_1}{100\%}\right)$  is the expected present value of benefit in the amount of 1 m.u. in the case of pensioner dies during the first month of retirement, reduced by the necessary expenditures  $\delta_1$  associated with the payment<sup>1</sup>, where

$$A_{x:\overline{1/12}|}^1 = \left. \frac{1}{12} \right| q_x \times P\left(\frac{1}{12}\right). \tag{7}$$

- The expression  $A_{x:\overline{1/12}|}^1 \times S \times \frac{\delta_1}{100\%}$  reflects expenditures of an insurance company associated with a quick withdrawal of finances from investment funds. We assume that the expenditures incurred in withdrawing money are the same as the expenditures associated with paying the amount to the heirs or an authorized person.
- The expression  $12 \times MP_x \times \left(IMA^{(12)}\right)_{x:\overline{7}|}^1 \times \left(1 - \frac{\delta_2}{100\%}\right)$  represents the expected present value of benefits which are intended for the heirs or an authorized person if pensioner dies during the first seven years, reduced by the necessary expenditures  $\delta_2$ .

The expression

$$\begin{aligned}
 \left(IMA^{(12)}\right)_{x:\overline{7}|}^1 &= \sum_{r=0}^6 \sum_{i=1}^{11} \left. \frac{12 \times r + i}{12} \right| \frac{1}{12} q_x \times \frac{12-i}{12} \times P\left(\frac{12 \times r + i + 1}{12}\right) \times \left(1 + \frac{j}{100\%}\right)^r + \\
 &+ \sum_{r=1}^6 \sum_{k=1}^{12 \times r} \left. \frac{k}{12} \right| \frac{1}{12} q_x \times \left(1 + \frac{j}{100\%}\right)^r \times P\left(\frac{k+1}{12}\right)
 \end{aligned} \tag{8}$$

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<sup>1</sup>It represents the situation under the Act, Article 46g, paragraph (5).

represents the expected present value of the sum of not yet paid monthly annuity in the amount of  $1/12$  of  $m. u.$  geometrically increasing by  $j\%$  per year, in the case of the beneficiary death during the period of the first seven years of pension payment<sup>2</sup>.

The expression  $12 \times MP_x \times (IMA^{(12)})_{x:\overline{7}|}^1 \times \frac{\delta_2}{100\%}$  expresses expenditures of the insurance company associated with a quick withdrawal of finances from investment funds. Even in this case we assume that the expenditures associated with withdrawing money are the same as the expenditures associated with paying the amount to the survivors or an authorized person.

- The expression  $12 \times MP_x \times \frac{\alpha}{100\%}$  represents lump sum initial costs from the first yearly annuity benefit.
- We denote initial costs as an absolute amount in monetary units by  $IC$ .
- The last expression  $12 \times MP_x \times \frac{\beta}{100\%} \times \sum_{t=1}^{12(\omega-x)} \frac{t}{12} | (\tilde{I}a)_x^{(12)}$  represents the expected present value of administrative costs, where the expected present value of deferred annuities for  $t = 1, 2, \dots, 12 \times (\omega - x)$ , is given by

$$\begin{aligned} \frac{t}{12} | (\tilde{I}a)_x^{(12)} &= (\tilde{I}a)_x^{(12)} - \frac{1}{12} \times \sum_{r=0}^{\lfloor \frac{t-1}{12} \rfloor} \sum_{s=1}^{12} I(t+1-s-12 \times r) \times \left(1 + \frac{j}{100\%}\right)^r \times \\ &\times \frac{12 \times r + s}{12} p_x \times P\left(\frac{12 \times r + s}{12}\right), \end{aligned} \quad (9)$$

where

$$I(z) = \begin{cases} 1 & \text{for } z > 0, \\ 0 & \text{otherwise.} \end{cases}$$

From equivalence equation (5) we get formula of the amount of monthly pension annuity  $MP_x$  of this product as follows.

$$MP_x = \frac{S \times \left(1 - A_{x:\overline{1/12}|}^1\right) - IC}{12 \times \left( (\tilde{I}a)_x^{(12)} + \frac{\alpha}{100\%} + (IMA^{(12)})_{x:\overline{7}|}^1 + \frac{\beta}{100\%} \times \sum_{t=1}^{12(\omega-x)} \frac{t}{12} | (\tilde{I}a)_x^{(12)} \right)}. \quad (10)$$

### 3.2 Geometric progression quotient

Product 2, as is defined in the Act, is a monthly paid annuity increasing yearly in geometric progression. The determination of the geometric progression quotient is a crucial task, especially when we consider calculation of the future cash-flows based on market-consistent valuation by risk-free bond yield curves.

In this part we point out the necessity of a mutual relationship between the risk free bond yields and the yearly increase rate of the pension payments.

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<sup>2</sup>It represents the situation under the Act, Article 32.

If we study an accumulated factor with progression quotient and also a discounting factor on the basis of the Svensson yield curve from formula (6), we get

$$\left(1 + \frac{j}{100\%}\right)^r \times P\left(\frac{12 \times r + i}{12}\right) = \left(1 + \frac{j}{100\%}\right)^r \times \exp\left\{-\frac{R\left(\frac{12 \times r + i}{12}\right)}{100\%} \times \left(\frac{12 \times r + i}{12}\right)\right\}.$$

After performing a technical modification, we obtain the expression

$$\frac{1}{\frac{\exp\left\{-\frac{R\left(\frac{12 \times r + i}{12}\right)}{100\%} \times \left(\frac{12 \times r + i}{12}\right)\right\}}{\left(1 + \frac{j}{100\%}\right)^r}}, \tag{11}$$

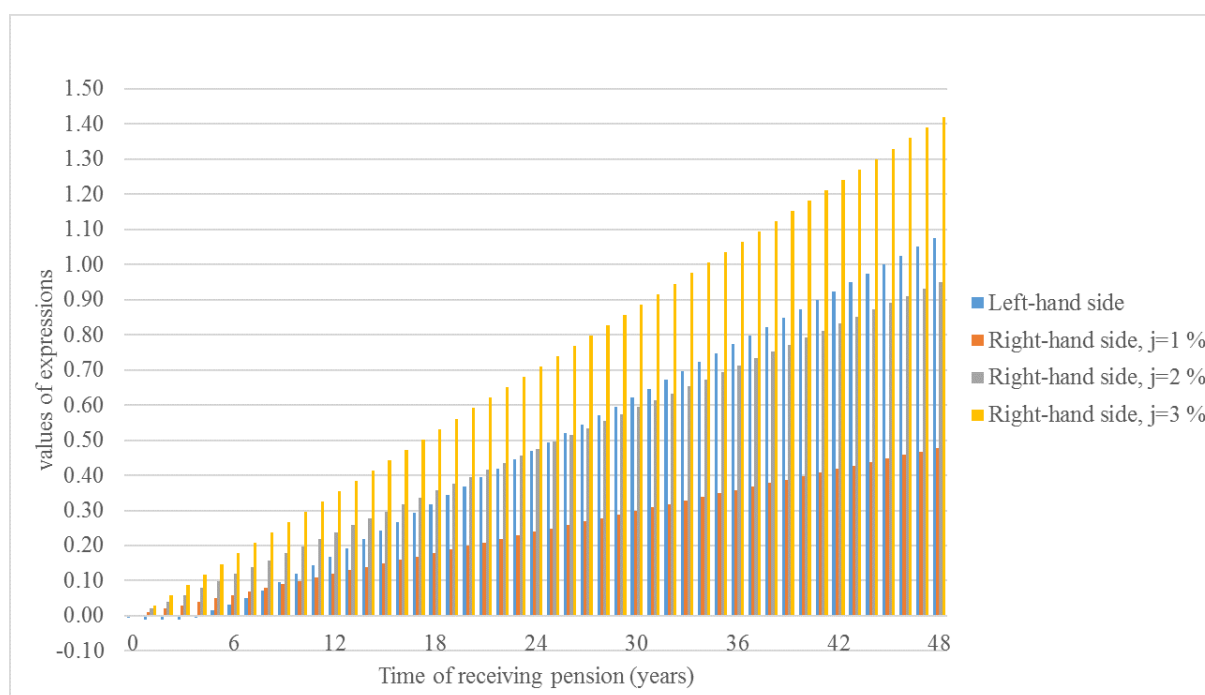
that represents the actual discount factor for individual financial flows. From a mathematical point of view, we may really consider positive capitalization, if the denominator of (11) is greater than one. So this condition is met if

$$\frac{R\left(\frac{12 \times r + i}{12}\right)}{100\%} \times \left(\frac{12 \times r + i}{12}\right) > r \times \ln\left(1 + \frac{j}{100\%}\right) \tag{12}$$

for all  $r = 0, 1, 2, \dots, \omega - x - 1$  and  $i = 1, 2, \dots, 12$  and a quotient  $j \geq 0$ .

Figure 1 illustrates the amounts of the right-hand side of inequality (12) for the geometric progression quotients gradually for  $j = 1, 2, 3\%$  and the amount of the left-hand side with yields of the Svensson yield curve<sup>3</sup>. Because in our next study we consider 62-age of retirement, we illustrate a time of receiving pension ranging from 0 to 48 years.

Figure 1: A comparison of the right-hand and left-hand side of inequality (12)



Source: The author's work.

<sup>3</sup>The Svensson yield curve on 7 December 2017, for AAA rated bonds (European Central Bank, 2017).

On the basis of individual amounts presented on Figure 1 we can consider for positive yield maximum progression quotient  $j = 1\%$ . However, we can investigate the impact of higher progression quotients what is shown in the next subsection. But these higher quotients cause negative yields from the investment money.

### 3.3 The amount of the monthly pension of Product 2

In this part we offer the amounts of monthly pension annuity of the Product 2. We recall that all amounts are calculated by formula (10) using the Lee-Carter model of longevity which was applied on mortality tables from years 1950-2014 with predictions on future period 2015-2063, for total population (The Human Mortality Database, 2017), linear interpolation between integer ages and the uniform distribution of deaths assumption and the Svensson yield curve on 7 December 2017, for AAA rated bonds (European Central Bank, 2017). Moreover, we consider the accumulated sum in the amount of 10,000 euros and expenditures  $\alpha = 20\%$ ,  $\beta = 0.1\%$  and  $IC = 300$  euros. In Table 1 there are listed increasing monthly pension annuities for 62-aged pensioner starting with the basic monthly pension in the amount of 33.02 euros gradually increasing by 1% per year up to his 110 years on the amount of 53.24 euros, if pensioner will be alive.

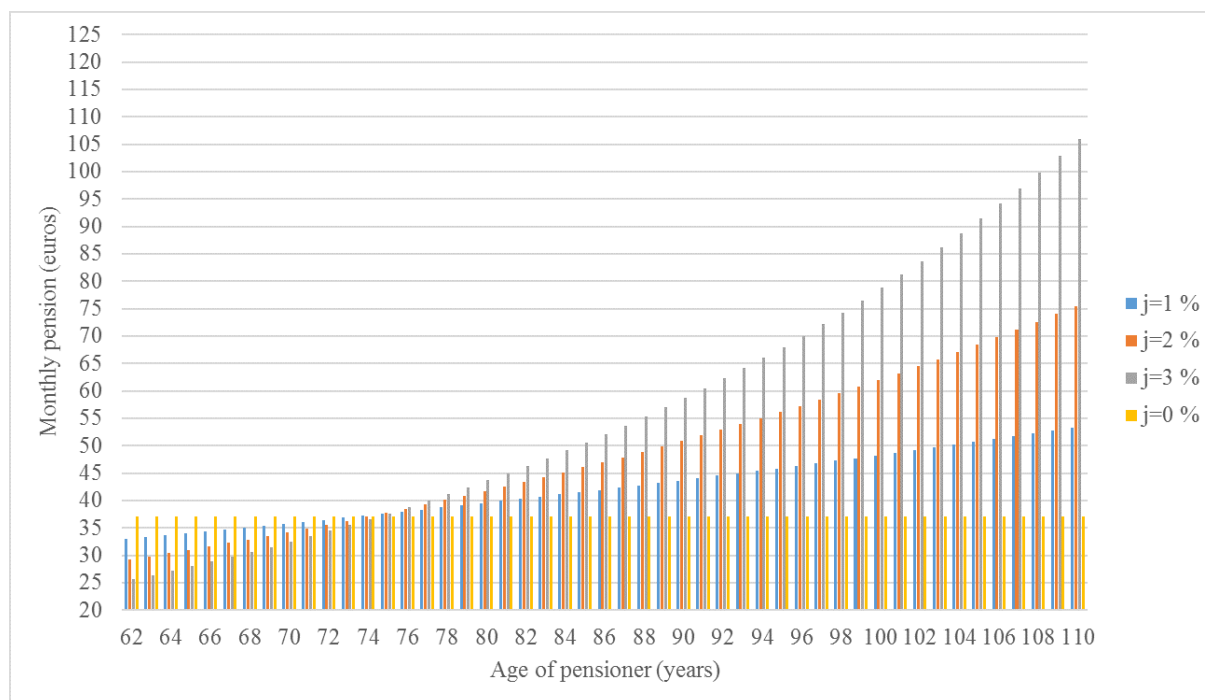
Table 1: The amounts of the monthly pension annuities geometrically increasing with  $j = 1\%$  for 62-aged pensioner,  $S = 10,000$  euros

payment year	monthly pension (euros)
1	basic pension 33.02
2	33.35
3	33.68
⋮	⋮
19	39.50
29	43.63
39	48.19
44	50.65
49	53.24

Source: The author's work.



Figure 2: Monthly pension annuities geometrically increasing by yearly progression quotient  $j$  for 62-aged pensioner with the accumulated sum 10,000 euros



Source: The author's work.

It is also very interesting to study the impact of higher progression quotient on the amount of the monthly annuities. This situation is illustrated on Figure 2. There are illustrated monthly pension annuities geometrically increasing gradually by  $j = 1, 2, 3\%$ . Moreover, there is also illustrated monthly pension annuity with  $j = 0\%$  which represents monthly pension of Product 1, for more details see (Špirková *et al.*, 2017).

Figure 2 shows how pension annuities evolve according to progression quotient. Pensioner can expect the same pension annuity like from Product 1 approximately after thirteen years. Although, for the first thirteen years he or she will have a lower pension, but in higher ages he can have much more higher pension, but if pensioner will be alive. This is more a philosophical or psychological problem than mathematical one.

#### 4. Conclusion

Our contribution offers the investigation of the monthly pension annuity of the old-age pension which is specified in the Act, in Article 46 as Product 2. By applying of the Council Directive 2004/113/EC of 13 December 2004 implementing the principle of equal treatment between men and women in the access to and supply of goods and services, Directive 2009/138/EC of the European Parliament and of the Council and Directive 2014/51/EU of the European Parliament and of the Council, we estimated the probability-weighted average of future cash-flows, taking account of the time value of money, using the Svensson yield curve of the December 7, 2017.

We focused on crucial role of the geometric progression quotient. On the basis of inequality (12) we currently recommend to offer increasing pension annuities with maximal progression

quotient in the amount of 1 %.

In the future we plan to continue in our detailed investigation of other products which are stated by the Act. Another issue that could be worth of our interest in future is annuity modelling according to Albrecher *et al.* (2016), Deprez *et al.* (2017), Konicz and Mulvey (2015). All calculations were made by the systems MS Office Excel 2016 and the R software (R Core Team, 2016), with package demography, (Hyndman *et al.*, 2014).

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