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ESTIMATES OF REGIONAL CAPITAL MATRICES: A CASE STUDY OF THE CZECH REPUBLIC

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Abstract

The paper is focused on the estimation of symmetric capital matrices by product in the Czech Republic. Symmetrisation is usually related to the intermediate consumption matrix, which is transformed from the dimension product by industry to the product by product matrix that is the starting point for input-output analysis. However, symmetrisation of capital matrix is very rare and has not been done in the Czech Republic. It is a very complex task as it is in fact double symmetrisation as original matrix of stocks of fixed assets is in the dimension type of asset (AN) by industry (NACE). Several data sources were applied the most important are supply and use tables, estimated transformation matrices. The results enable sophisticated input-output analysis focused on the capital and the demand for capital that is caused by economic impulse or shock. Finally, regionalization of the national results is performed by using additional data and RAS procedure. The regional results are compared with other regional economic indicators estimated within the previous research. It extends a list of regional indicators available for analysis and modelling on the regional level.

***Key words**: Capital matrices, Symmetrisation, Capital formation, Fixed assets*

***JEL Codes**: E22, D57, C67*

1. Introduction[1]

Capital matrices play important role in economic analysis. They are used in various economic models and applications. Namely, capital matrices are used within the Input-Output Analysis in dynamic models (Leontief, 1953, 1970), environmental applications or in disaster evaluation models (Okuyama, Lim, 2002). They have their own place in Computable General Equilibrium models (CGE) and in Dynamic Stochastic General Equilibrium models (DSGE), as well. In the structural analysis (as is I-O models or CGE models) it is important to have variables in the same classifications. The preferred classification is the product classification such a CPC (central product classification) or CPA (classification of products by activity). However, official statistics provides very little data on capital broken down by the product classification since the main purpose is to describe who (industry) invests (or owns) which type of assets. These capital matrices are usually broken down by industrial classification (ISIC or NACE) in columns and type of assets that is purchased, sold or owned in rows. The

[1] Our paper and estimations are based on main author‘s dissertation (Šafr, 2018). Dissertation defence is scheduled in September 2018.

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product breakdown is applied only within supply and use tables framework in which gross fixed capital formation has to be split into products. In fact, data in 'type of asset' division can be transformed to product classification using additional data sources and assumptions. The term 'type of asset' does not refer to any product classification as assets may be either non-financial or financial but non-financial assets only can be classified to the product classification. Capital is limited to non-financial assets in the paper as only investments to non-financial assets belong the goods and service accounts consequently gross domestic product (GDP).

As mentioned above, many econometric models require identical breakdown for all variables. Capital matrices are not widely used by researchers since they are not available. One aim of the paper is to transform capital matrices into symmetric input-output tables (product by product). It should be clarified that the term "symmetrisation" in our paper means that we symmetrise column's and row's classification, not column's and row's sums – as done in the Input-Output Analysis. Our approach is based on the stepwise re-classification of the dataset from matrices in "Type of assets" (rows) and NACE (columns) classification to matrices in CPA (rows) and NACE (columns) classification. Finally, they are transformed to product by product tables.. The first part of the paper is devoted to literature review and methodology and the second part describes the estimation procedure and the results.

2. Literature review

The term 'symmetric matrix' may be confusing in input-output tables field and should be defined at the beginning. It does not refer to symmetric matrix from mathematical point of view, but it means that the same classification is used in rows and columns of a matrix. It is obvious that they have to be square: the number of rows is equal to the number of columns. Symmetric input-output tables represent a powerful analytical tool that is used in productivity, energy or environmental analysis (Eurostat, 2008). These tables are not complied directly but transformed from supply and use tables, which describe resources and uses of products in national economy, using various assumptions. The choice of assumption and consequently a model depends on the classification (product or industrial) and other conditions. The aim of the paper is to transform capital matrices to product by product tables. The transformation to product input-output tables can be done using product technology assumption (model A in the Manual) or industry technology assumption (model B in the Manual). Model A is preferred to Model B (Eurostat, 2008). Model A assumes that each product is produced in its specific technology regardless the industry produces that product. The detailed description of each transformation can be found in Eurostat (2008) or Vavrla, Rojíček (2006).

Regarding capital matrices, estimates can be made by several approaches. These approaches are divided into three groups of methods:

- Methods based on detailed surveys. These surveys have to be able to identify the original and final destination of capital.
- Methods based on model estimation – i.e., calibration from dynamic models and other (Šafr, 2016).
- Hybrid methods that combine both above mentioned approaches.

As many authors state (Pauliuk *et al.*, 2015) the effect of technical capital matrices in structural models is not widely described area. However, the topic of capital formation/stock is discussed in many works (OECD, 2009). The basic knowledges describe Leontief (1970, 1985) and Thijs tee Raa (1986). They are explaining the area of capital matrices in Input-Output systems and models.

Regionalization approaches is big topic in Input-Output analysis due to inaccessibility of regional matrices. Generally, these matrices are calculated on the basis of intermediate use. The approaches for regionalization can be separated by survey methods, model estimations and hybrid techniques. One of the most used method is the location quotients that can be run on several data sources and by several methods (Flegg *et al.*, 1995). Above that, RAS and CE methodology are applied in several applications.

3. Methodology

There are several problems connected with estimating capital matrices at regional level. The first major obstacle is that typically the national structure of capital is monitored fixed assets classification. Generally, non-financial assets (AN) are divided into assets produced (AN.1) and non-produced assets (AN.2). Produced assets are further divided into fixed assets (AN.11), inventories (AN.12) and valuables (AN.13). The paper is focused on fixed assets that represent the main capital input in production process. They are further divided into tangible fixed assets (AN.111) and intangible fixed assets (AN.112). Gross capital formation is a single capital indicator available in product classification which necessary for symetrisation. Second problem is that the classification of stocks and GFCF (in AN classification) is not directly transformable into the classification of production (CPA) or industry (NACE). Thus, according to this classification, the originator of the capital resources cannot be directly identified and it is necessary to reclassify the fixed asset classification (AN) to the classification by product or industry.

Other problems linked to capital matrices in Input-Output analysis are caused mainly by instability of technical coefficients. This problem arises especially in model-based approaches. Assumption of stability of technical coefficients is crucial in Input-Output analysis. Nonetheless, stability of technical coefficients of capital matrices is highly problematical (e.g. Miller and Blair, 2009). Calibration based on dynamic Input-Output model showed twice-larger estimates than is observed by statistical offices (e.g. Šafr, 2016). The difference between stability of technical coefficients of intermediate use matrices and capital matrices lies in several areas:

- Relationship between investment and total output is not direct as in case of intermediate use.
- Capital formation does not have to be continuous in short term (1-5 years, e.g. cars, high-tech products etc.).
- The recovery of capital in specific sectors may not be smooth even over a long time period (5-10years, e.g. building)

Because of these problems, we focus on reclassification of known figures. This reclassification is conducted in three-stage process. In first stage the known figures of type of assets (type of assets x NACE) are transformed into CPA x NACE matrices. In second stage, we transform these matrices into CPA x CPA matrices. Finally, in last stage the regional figures are estimated (in CPA x CPA) by modified three stage RAS.

3. 1 Used data

We focus on estimating capital matrices for Czech Republic for the year 2013. Publicly available data does not cover capital matrices in sufficient level of detail. Due this reason, we are using internal data from Czech statistical office – capital matrices in type of assets in rows and industries in columns (21 rows x 88 columns). These data cover capital formation and capital stock (AN.11). Publicly available data covers only 10 types of assets and 21 industries. In the second stage, we use a Supply and Use tables – which is publicly available from the Czech Statistical Office. These tables are generally used for balancing and deflation of GDP by production and expenditure approach. They also serve as the starting point for the compilation of input-output tables.

In the third stage (regionalization), we use inter-regional matrices (Department of Economic Statistics, 2018). These matrices were estimated by hybrid methodology – which combines several data sources and model estimation (Sixta, Vltavská, 2016; Šafr, 2016). These data are combined with publicly available figures of regional sums of capital stock (Czech Statistical Office, 2018).

3. 2 Estimates on the national level

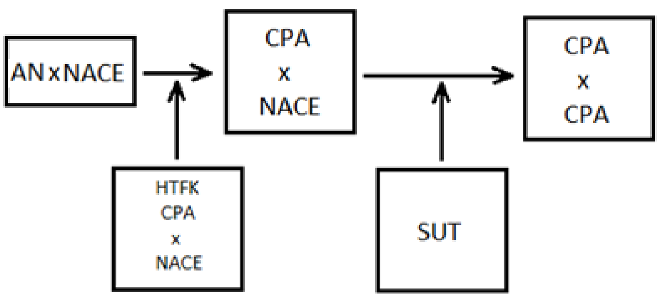
The transformation of allocation of the capital formation is prepared in the same way as for the capital stock. It is similar to the procedures used by the Czech Statistical Office. We apply these rules on capital stocks for year 2013. The biggest disadvantage at this stage is the assumption of the same structure of flows represented by gross fixed capital formation and net stocks. In fact, stocks have been created by investments in precedent years. It should be mentioned that structures of investment can change over time. Following indicators are affected by the transformation: Consumption of fixed capital, Nominal holding gains/losses, other changes in volume. The transformed matrix has CZ-CPA classification in rows and CZ-NACE classification in columns. In other words, it can be found in the matrix which capital product is used in production process in which industry.

In second stage, we transform this matrix into CPA x CPA matrix. This transformation is done using supply and use tables (SUT). The symmetrisation is carried out by based on the product technology assumption. We follow the Eurostat Manual of Supply, Use and Input-Output tables in this procedure (Eurostat 2008), but we have to make some necessary adjustments:

- we do not calculate with product which is not in original CPAxNACE matrix.
- UT are not used directly but average structure of SUT for the period 1995 – 2013 is applied due to stability of data.
- Inconsistency is solved within RAS procedure.

Possible drawback in this step lies in the assumption of same allocation of capital stock by average allocation of intermediate use. We assume that this step will not affect our matrix significantly and hope that will not be bigger than 20% (absolute percentage distance between matrices). The numbers higher more than 20% will be hardly understandable. The overall procedure can be visualized on following diagram:

Figure 1: Procedure of estimating of Capital matrices



Source: The author's work

3. 3 Regionalization

The procedure of regionalization is highly experimental and is based on known figures of capital formation by products (CPA). The second source of data is Inter-regional input-output tables for the year 2013 and our own national estimates of capital stocks in products (CZ-CPA classification). We have to balance this data in the final stage of estimation.

The known figures of regional products stock (CPA) are used as structure bounds of regional submatrices. The Inter-regional matrix is used as leading structure of more detailed level of data than is in regional product stocks. The national figures are used as bounds for national sums (trough regions). We can summarize this in following equations:

Regional Capital formation matrix (for R-regions):

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & \cdots & \mathbf{C}_{1R} \\ \vdots & \mathbf{C}_{pr} & \vdots \\ \mathbf{C}_{R1} & \cdots & \mathbf{C}_{RR} \end{bmatrix} \tag{1}$$

Should hold known figures of capital formation (regional consistency):

$$\mathbf{C}_{pr} = \begin{bmatrix} c_{ij}^{pr} & \cdots & c_{ij}^{pr} \\ \vdots & c_{ij}^{pr} & \vdots \\ c_{ij}^{pr} & \cdots & c_{ij}^{pr} \end{bmatrix} \tag{2}$$

$$\mathrm{a}_l^p = \sum\nolimits_{r=1}^{R} \sum_{i=i_1^l}^{i_3^l} \sum_{j=i_1^1}^{i_2^l} c_{ij}^{pr} \text{, for all } l, p. \tag{3}$$

Where a_l^p is known regional aggregation of l-th capital formation and i_1^l, i_2^l, i_3^l just its aggregation field.

Second aggregation condition arise from national sums of each element of matrix **C** to national matrix $\mathbf{C}^{\mathrm{N}}$:

$$c_{ij}^{\mathrm{N}} = \sum\nolimits_{p=1}^{P} \sum\nolimits_{r=1}^{R} c_{ij}^{pr} \text{, } for\ all\ i, j. \tag{4}$$

Finally, the next condition comes from structure distance minimization. This condition can be defined simply as:

$$\min w = \sum_{p=1}^{P} \sum_{r=1}^{R} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{c_{ij}^{pr}}{\sum_{p=1}^{P} \sum_{r=1}^{R} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^{pr}} \log \left(\frac{\dfrac{c_{ij}^{pr}}{\sum_{p=1}^{P} \sum_{r=1}^{R} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^{pr}}}{\dfrac{x_{ij}^{pr}}{\sum_{p=1}^{P} \sum_{r=1}^{R} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}^{pr}}} \right) \right) \tag{5}$$

Where x_{ij}^{pr} is inter-regional flow of intermediate consumption. This equation is Kullback-Leibler distance or Cross entropy formulation of minimal distance. This problem (eq -) can be solved by Cross entropy or adequately same by modified three stage RAS method (or multidimensional RAS approach).

4. Results

Following figures summarize our estimates at the most detailed regional level. Estimated capital stocks in each region's products (CPA) are shown in table 1. The interpretation of the results in Table 1 is a little bit complicated. It describes capital stock in regions (column) used in production of products (rows). The highest stock is observed in Prague region of which the highest part in production of real estate services (CPA L). It is noteworthy that the highest share of CPA L on total capital stock can be seen in all regions. However, the bigger one (23.3%) is in Prague region as real estate services including services are important product in Prague region. Besides, prices level of dwelling services is higher as a result of expensive dwellings (Čadil *et al.*, 2014). On the other hand, the lowest share (10.3%) of product C (manufactured products) is observed in the capital city as these products are not produced there. Almost 40% of capital in Středočeský region is used in production of industrial goods (CPA C) which is headquarters of important car producers and their suppliers.

Table: 1: Capital stocks in each CPA and Region (Bil. CZK)

	Jhc	Jhm	Kar	Krh	Lib	Mrs	Olm
A	100	130	28	57	38	62	58
B	44	31	16	10	10	54	11
C	340	695	91	394	302	964	350
D	206	143	75	46	46	254	49
E	23	16	9	5	5	29	6
F	49	119	20	28	28	73	35
G	63	145	25	49	56	106	69
H	123	285	49	97	111	209	137
I	14	33	6	11	13	24	16
J	60	168	25	40	37	94	48
K	23	51	6	15	12	38	17
L	221	658	102	174	154	405	220
M	33	101	12	23	30	57	28
N	32	101	11	21	32	55	26
O	143	364	60	112	100	269	152
P	62	157	26	48	43	116	65
Q	22	56	9	17	15	41	23
RST	19	89	6	11	11	51	17

	Par	Pha	Plz	Stc	Ust	Vys	Zln
A	70	92	80	138	56	100	46
B	23	66	18	38	66	13	11
C	364	774	364	1543	439	371	499
D	106	311	85	178	308	60	52
E	12	35	10	20	35	7	6
F	40	308	43	89	47	38	43
G	50	362	63	166	73	45	56
H	99	713	124	327	143	89	110
I	11	82	14	37	16	10	13
J	65	878	72	112	50	54	50
K	22	454	16	60	20	15	27

L	204	1756	176	648	252	211	287
M	27	378	31	110	33	28	32
N	25	423	33	114	30	26	26
O	106	492	136	228	146	86	124
P	45	212	59	98	63	37	53
Q	16	76	21	35	22	13	19
RST	13	121	16	45	21	10	22

Source: The author's work

Following table shows capital stocks in each region sorted by the highest values for types of CPA. On other words, it describes which products are stored in stocks. The prevailing products are buildings and other structures (CPA F) in all regions. The highest share of research and development (CPA J) is observed (not surprisingly) in Prague region.

Table 2: Which CPAs is in which regions in capital stock (Bil. CZK)

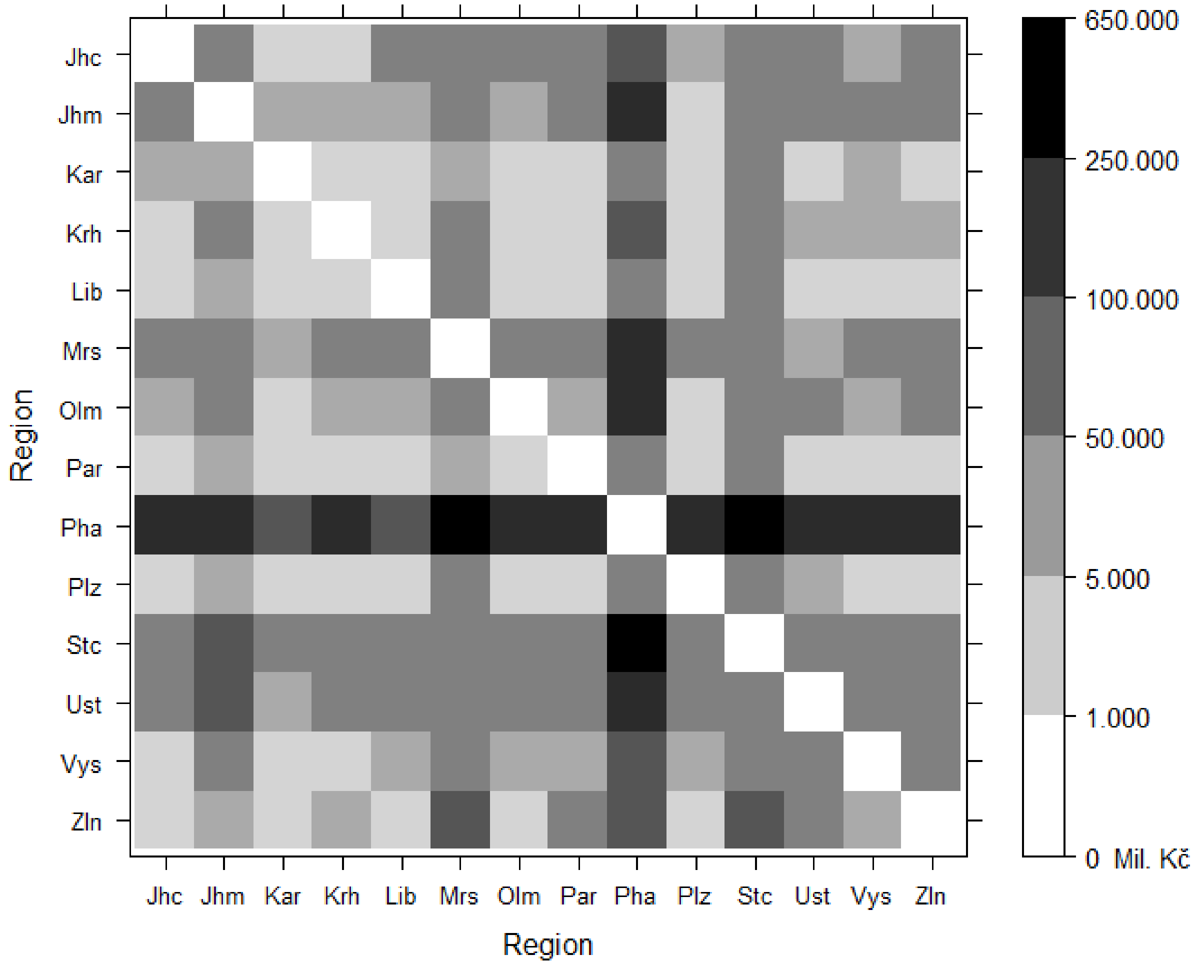
	Jhc	Jhm	Kar	Krh	Lib	Mrs	Olm
A	1	2	0	1	1	1	1
C	298	595	98	235	207	589	244
F	1238	2648	462	890	805	2229	1046
J	14	36	5	10	9	25	12
M	25	60	7	24	22	58	23

	Par	Pha	Plz	Stc	Ust	Vys	Zln
A	1	2	1	2	1	1	1
C	257	1314	271	865	348	247	291
F	1003	5892	1048	2987	1428	928	1139
J	14	176	14	34	14	12	13
M	23	148	26	97	28	24	30

Source: The author's work

Next picture shows capital stocks splited by origin and final destination. You can see that the highest outcome and income of capital are provided by Prague region. Prague region supply one of the highest capital stock in each region (sometimes second highest).

Figure 2: Structure of regional capital matrix for year 2013 at current prices (mil. CZK)



Source: The author's work

5. Conclusion

We presented methodology of estimating capital matrices and we showed hat it is possible to estimate them at regional level (and in same dimension and classification as input-output tables). The main advantage of this approach is that no additional survey is needed. However, the methodology is based on strong assumptions, which are connected with use of the capital stocks that have to follow the structure of the intermediate use. On regional level, the assumption is powered by relative structure of the intermediate use; on other hand, the impact is bounded by known figures of capital formation on "sub-regional" level.

The results proved that economy of Prague region is specific and different to other regions' economies. Capital is used to the greatest extend in the production of real estate services. In other regions, capital is employed mainly in the production of manufacturing goods. Prague is also the center of research activities, which is confirmed by the highest stock of the asset.

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Appendix

Table 3: Abbreviations and names of Czech regions

Abbreviation	Full name
Jhc	South Bohemian Region
Jhm	South Moravian Region
Kar	Karlovy Vary Region
Krh	Hradec Králové Region
Lib	Liberec Region
Mrs	Moravian-Silesian Region
Olm	Olomouc Region
Par	Pardubice Region
Pha	Prague
Plz	Plzeň Region
Stc	Central Bohemian
Ust	Ústí nad Labem Region
Vys	Vysočina Region
Zln	Zlín Region

Table 4: Abbreviations and names of CPA products

Abbr.	Statistical Classification of Products by Activity
A	PRODUCTS OF AGRICULTURE, FORESTRY AND FISHING
B	MINING AND QUARRYING
C	MANUFACTURED PRODUCTS
D	ELECTRICITY, GAS, STEAM AND AIR CONDITIONING
E	WATER SUPPLY; SEWERAGE, WASTE MANAGEMENT AND REMEDIATION SERVICES
F	CONSTRUCTIONS AND CONSTRUCTION WORKS
G	WHOLESALE AND RETAIL TRADE SERVICES; REPAIR SERVICES OF MOTOR VEHICLES AND MOTORCYCLES
H	TRANSPORTATION AND STORAGE SERVICES
I	ACCOMMODATION AND FOOD SERVICES
J	INFORMATION AND COMMUNICATION SERVICES

K	FINANCIAL AND INSURANCE SERVICES
L	REAL ESTATE SERVICES
M	PROFESSIONAL, SCIENTIFIC AND TECHNICAL SERVICES
N	ADMINISTRATIVE AND SUPPORT SERVICES
O	PUBLIC ADMINISTRATION AND DEFENCE SERVICES; COMPULSORY SOCIAL SECURITY SERVICES
P	EDUCATION SERVICES
Q	HUMAN HEALTH AND SOCIAL WORK SERVICES
R	ARTS, ENTERTAINMENT AND RECREATION SERVICES
S	OTHER SERVICES
T	SERVICES OF HOUSEHOLDS AS EMPLOYERS; UNDIFFERENTIATED GOODS AND SERVICES PRODUCED BY HOUSEHOLDS FOR OWN USE
U	SERVICES PROVIDED BY EXTRATERRITORIAL ORGANISATIONS AND BODIES