ANALYSIS OF POTENTIAL MARRIAGE REVERSE ANNUITY CONTRACTS BENEFITS IN SLOVAK REPUBLIC

AGNIESZKA MARCINIUK
Wroclaw University of Economics, Faculty of Management, Computer Science and Finance,
Department of Statistics,
ul. Komandorska 118/120, Wrocław, Poland
e-mail: agnieszka.marciniuk@ue.wroc.pl

EMÍLIA ZIMKOVÁ
Matej Bel University, Faculty of Economics,
Department of Finance and Accounting,
Tajovského 10, Banská Bystrica, Slovakia
e-mail: emilia.zimkova@umb.sk

Abstract

Demographic trends in Europe reveal that the pension funding gap will become one of the key social issues in coming years. On the other hand, many of these people hold a large amount of wealth in their property which, being reasonably utilized by equity release products, could help them cover their needs. People can surrender their real estate to a company interested in the acquisition of their property in exchange for the whole life monthly benefits. The aim of the contribution is to analyze the potential benefits of marriage reverse annuity contracts in the Slovak Republic by the use of the Svensson model function, considering the reversionary annuity and the real value of estate in different cities. The real value of the properties is determined by the place where spouses live and has significant influence on the amount of the benefit. While in many countries the equity release products have been offered to clients for dozens years, a product of this nature has not been established so far in the Slovak Republic. Hopefully, this contribution might initiate discussions on the introduction of similar equity release products market in the Slovak Republic as well.

Keywords: *reverse annuity contract, reverse mortgage, reversionary annuity*

JEL Codes: *C41, C60, G17, G22, G120, J1.*

1. Introduction

A very big progress in the development of medicine and the growing awareness within the society of healthy nutrition and lifestyle contributes to the increase in life expectancy, which has been increasing in almost all the countries of the world (cf. Blake *et al.*, 2013). In the last sixty years, an increase in life expectancy of about 11 years has been observed.

The European silver aged population is increasing. This population group will probably not have enough income to cover their retirement needs, because social insurance pensions and incomes are low. High bills for utilities and rent, and also for medical care and medicines could be an important problem especially in big cities or in a situation where one of the spouses dies.

In this context, a very important issue is the possibility of obtaining additional financial resources. Many people hold a large amount of wealth in their property, but most of them are

reluctant to sell their properties and relocate. Owners can surrender their real estate to a company interested in the acquisition of their property in exchange for the whole life monthly benefits.

The main two types of equity release products are loan model (reverse mortgage scheme) and sale model (home reversion scheme). Different varieties of these products exist in many countries (cf. Hanewald *et al.*, 2016; Charupat, *et al.*, 2016). We focus on two contracts, that is the reverse annuity contract (a sale model) and reverse mortgage (a lone model) that exist in an individual form in Poland (cf. Marciniuk, 2017).

Since in many cases the property owners are couples, an important issue is enabling the marriage reverse annuity contract or marriage reverse mortgage when both spouses are alive, and when one of them dies. We apply the reversionary annuity to calculate the benefit in the Slovak Republic. We distinguish various cases of such products depending on the percentage value of annuity, which is received after the death of the one spouse. The benefits depend on the age of the spouses, their future lifetime and the real value of their properties which, in turn, determines the place where they live. The real value of their properties has significant influence on the amount of the benefit. Therefore, the aim of this paper is to calculate the annuities for some regions of Slovakia on the basis of the real Slovak data from 2014 (www.mortality.org) and 2017 (the National Bank of Slovakia and www.reality.sk). These calculations are based on the real interest rate function depending on time *t* for Svensson model, which are disclosed at the website of the European Central Bank (https://www.ecb.europa.eu). Finally, the results are discussed and compared.

2. Benefit's formulas

In this paper we concentrate on a marriage reverse annuity contract and a marriage reverse mortgage, which are variations of individual reverse annuity contract. However, we also distinguish other indirect cases. Under these contracts, annuity benefits are payable when both spouses are alive and sometimes after the death of whichever spouse. Thus we distinguish between two types of such contracts: a *Joint-Life Status contract*, when the benefit is paid only until the death of the first spouse and a *Last Surviving Status contract* by which the benefit is paid until the death of the other spouse.

We distinguish the contract which pays yearly 1 financial unit as long as both members are alive and a fraction *R* of it (*R* means a reduction factor, $R \in [0,1]$) when only one member of the couple is alive. In this scheme, when $R = 1$ means the Last Surviving Status (the benefit paid remains constant also after the death of the first spouse), and the Joint-Life Status corresponds to $R = 0$ (nothing is paid to the last survivor). We also consider other cases, when *R* is other than 0 or 1, i.e. $R \in \left\{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right\}$ (cf. Luciano, *et al.*, 2016).

Let K_x and K_y denote the future lifetimes of *x*-year-old husband and *y*-year-old wife. Let ω_x (resp. ω_y) denote the difference between the age limit ω of the man (resp. woman) and man's (resp. woman's) age at entry *x* (resp. *y*). The benefit of reverse annuity contract is paid for the whole life, and reverse mortgage is paid only for *n*-years. Note that according to Life Tables the age limit $\omega = 100$ years or $\omega = 100$ (we use Life Table for $\omega = 100$). This implies that the maximum possible duration of the marriage reverse annuity contract is equal to $\max\{\omega_x, \omega_y\}$.

A *Joint-Life Status* (JLS) is defined as follows (cf. Bowers, *et al.*, 1986):

$$u := x : y$$

A future lifetime of this status is denoted by

$$K_u = \min(K_x, K_y) \tag{1}$$

The probability that status u will be surviving for at least k years is calculated by the following formula:

$${}_k p_u = {}_k p_{x:y} = P(K_u \geq k) = P(K_x \geq k, K_y \geq k) \tag{2}$$

where $k \in \{0,1,...,\min\{\omega_x, \omega_y\}\}$.

A *Last Surviving Status* (LSS) is denoted and defined by the use of w, i.e. (cf. Bowers, *et al.*, 1986):

$$w := \overline{x:y}.$$

A future lifetime of status w corresponds to K_w and is defined as a maximum of K_x and K_y, i.e.

$$K_w = \max(K_x, K_y) \tag{3}$$

The probability that status w will be surviving for at least k years is calculated by the use of ${}_k p_{x:y}$ as follows

$$\begin{aligned} {}_k p_w = {}_k p_{\overline{x:y}} &= P(K_w \geq k) = P(K_x \geq k \vee K_y \geq k) = \\ &= P(K_x \geq k) + P(K_y \geq k) - P(K_x \geq k, K_y \geq k) = \\ &= {}_k p_x + {}_k p_y - {}_k p_{x:y}, \end{aligned} \tag{4}$$

where $k \in \{0,1,...,\max\{\omega_x, \omega_y\}\}$.

We calculate the benefit of reverse annuity contract by the use of the reversionary annuity, on the basis of **Lemma 1** and the benefit of marriage reverse mortgage on the basis of **Lemma 2** for $m = 1$ (cf. Marciniuk, 2017), which can be written by the use of the following corollary.

Corollary

The yearly benefit of marriage reverse annuity contract for spouses (x, y)*, which pays 1 at the beginning of a year as long as both members are alive and* R $(R \in [0,1])$*, when only one member of the couple is alive, is calculated as follows*

$$\ddot{b}_{(x,y)} = \frac{\alpha W}{R\ddot{a}_x + R\ddot{a}_y + \ddot{a}_{x:y}(1-2R)}, \tag{5}$$

where

$$\ddot{a}_{x:y} = \sum_{k=0}^{\infty} v^k \, {}_k p_{x:y} \tag{6}$$

$$\ddot{a}_x = \sum_{k=0}^{\infty} v^k \ {}_kp_x \tag{7}$$

The n-term yearly benefit of marriage reverse mortgage value of due life annuity for spouses (x, y)*, which pays 1 financial unit at the beginning of a year as long as both members are alive and a fraction R of it when only one member of the couple is alive, is calculated as follows*

$$\ddot{b}_{(x,y):\,\overline{n}|} = \frac{\alpha W}{R\ddot{a}_{x:\,\overline{n}|} + R\ddot{a}_{y:\,\overline{n}|} + \ddot{a}_{x:y:\,\overline{n}|}(1-2R)}, \tag{8}$$

where

$$\ddot{a}_{x:y} = \sum_{k=0}^{n-1} v^k \ {}_kp_{x:y} \tag{9}$$

$$\ddot{a}_x = \sum_{k=0}^{n-1} v^k \ {}_kp_x \tag{10}$$

Moreover W *is the real value of benefit,* α *is the percentage of* W $\left(\alpha \in (0\%, 50\%]\right)$.

Generally α and W can be different. The benefit for the other parameters for example W_1 and α_1 can be calculated from the following formula

$$\ddot{b}_1 = \frac{\alpha_1 W_1}{\alpha W} \ddot{b} \tag{11}$$

where $\ddot{b}$ is given by (5) or (8).

Obviously, it is possible to assume that $\alpha = W = 1$, but usually $\alpha = 50\%$ and $W \neq 1$. It is easier to see the differences between the value of benefits. Therefore in section 5 we assume that $\alpha = 50\%$ and W follows from the Slovak market.

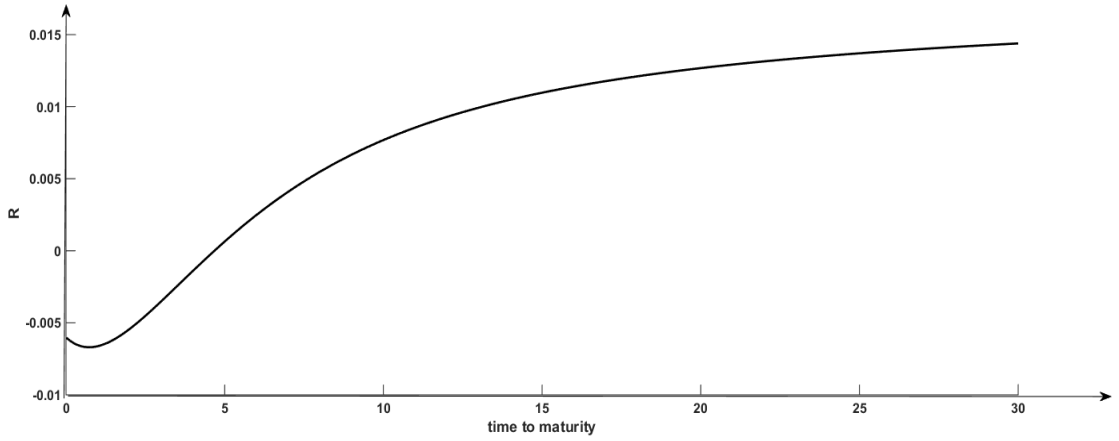
3. Rate of interest

The discounting factor v^k for $k = 1, 2, ..., n$ is given by the use of function $R_{0,k}$ as follows

$$v^k = \exp\left(-k \cdot R_{0,k}\right). \tag{12}$$

The parameters of function $R_{0,k}$ followed from the European Central Bank 5.02.2018 (cf. European Central Bank, 2018). This bank gives information about the data, the best fitted model of spot interest $R_{0,k}$ and its parameters for the euro zone. It is updated every TARGET business day at noon (12:00 CET). The Svensson model of spot interest rate is applied which is presented in Figure 1.

Figure 1. The model of spot interest rate



Source: own research on the basis of European Central Bank.

In case of Svensson model, function $R_{0,k}$ has the following form (cf. De Rezende and Ferreira, 2013; Anderson, *et al.* 1996)

$$R_{0,k} = \beta_0 + \beta_1 \frac{\tau_1}{k}\left(1-e^{-\frac{k}{\tau_1}}\right) + \beta_2\left(\frac{\tau_1}{k}\left(1-e^{-\frac{k}{\tau_1}}\right)-e^{-\frac{k}{\tau_1}}\right) + \beta_3\left(\frac{\tau_2}{k}\left(1-e^{-\frac{k}{\tau_2}}\right)-e^{-\frac{k}{\tau_2}}\right), \qquad (13)$$

Where $\beta_0 \geq 0, \beta_0 + \beta_1 \geq 0, \tau_1, \tau_2 > 0.$
The parameters are the following

$\beta_0 = 0.01781,\ \beta_1 = -0.02382,\ \beta_2 = 0.24034,\ \beta_3 = -0.26857,\ \tau_1 = 2.15465,\ \tau_2 = 2.11922.$

Parameter β_0 is the long term rate.

4. Location

The place where people live determines the real value of properties. We chose eight cities in different parts of the Slovak Republic. The price (in euro) per square meter of an apartment follow from secondary market at the end of 2017 (cf. www.reality.sk). The data was obtained for bachelor flats $(0, 32\ m^2]$, one-room flats $(0, 36\ m^2]$, two-room flats (about 60 m^2), three-room flats (about 75 m^2), four-room flats (about 110 m^2) and five-room flats (below 110 m^2). The size of the apartments was divided into three groups (cf. Marciniuk, 2017). The bachelors flats and one-room flats form one group of properties, which size is up to $36\,m^2$. The second group is flats of size from interval $(36\ m^2, 60\ m^2]$. The three-, four- and five-room flats were combined into one group, which size of an apartment is from 60 m^2 to 110 m^2. We calculated also the average price as an arithmetic mean of all available prices in each city. These prices in euro are presented in Table 1.

Table 1: The price (in euro) per square meter of an apartment in Slovakia

size of apartment m^2	**(0, 36]**	**(36, 60]**	**(60, 110]**	**different**
	price per square meter			average price
Banská Bystrica	1645	1441	1259	1334
Bratislava	**2271**	**2243**	**2042**	**2207**
Košice	1682	1530	1393	1565
Nitra	1939	1365	1271	1581
Prešov	1258	1139	1120	1263
Trenčín	1411	1274	1181	1320
Trnava	1558	1418	1266	1503
Žilina	1554	1351	1229	1346

Source: own research on the basis of the National Bank of Slovakia and www.reality.sk.

The largest prices are in capital. Regardless of the size of the apartments, the prices in Bratislava are similar in value. The highest prices per square meter are for small apartments. In Nitra, the small flats are quite expensive. The other are cheaper. In other cities prices are smaller and the differences between prices are not so significant. The cheapest apartments are in Prešov, almost twice cheaper than in Bratislava.

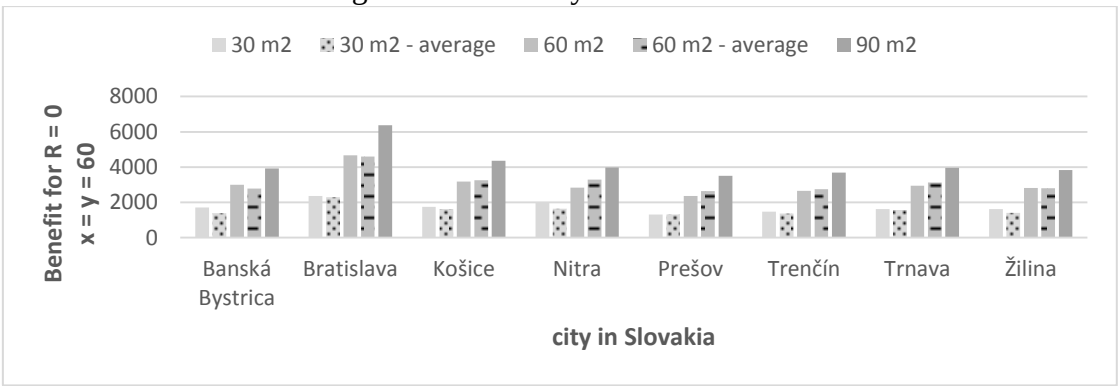
Cities can be divided into three groups due to the average price (cf. Table 1). The most expensive apartments are in Bratislava (above 2000 euro). Medium prices form interval [1400 euro, 2000 euro] are in Nitra, Košice and Trnava. The cheapest apartments are in Žilina, Banská Bystrica, Trenčín and Prešov (below 1400 euro).

5. Numerical examples

In this section the results of numerical calculation on Slovak real data are presented. All calculations are made using own programs written in MATLAB. We assume that $\alpha = 50\%$.

First, the yearly benefits of marriage annuity contract are presented in Figure 2 for a married couple, when wife and husband are at the same age $x = 60$. It is assumed that the marriage has a thirty, sixty or ninety square meter apartment. Its real value depends on the spouses' place of residence (cf. Table 1). We assumed that the life duration of spouses are independent random variables. The Slovak Life Tables follows from 2014 from www.mortality.org. We calculate the benefit marriage reverse annuity contract for $R = 0$ in different cases under the assumption that the price is different for various size of an apartment or the average price (by the use of price from last column in Table 1).

Figure 2. The benefit of marriage reverse annuity contract for $R = 0$

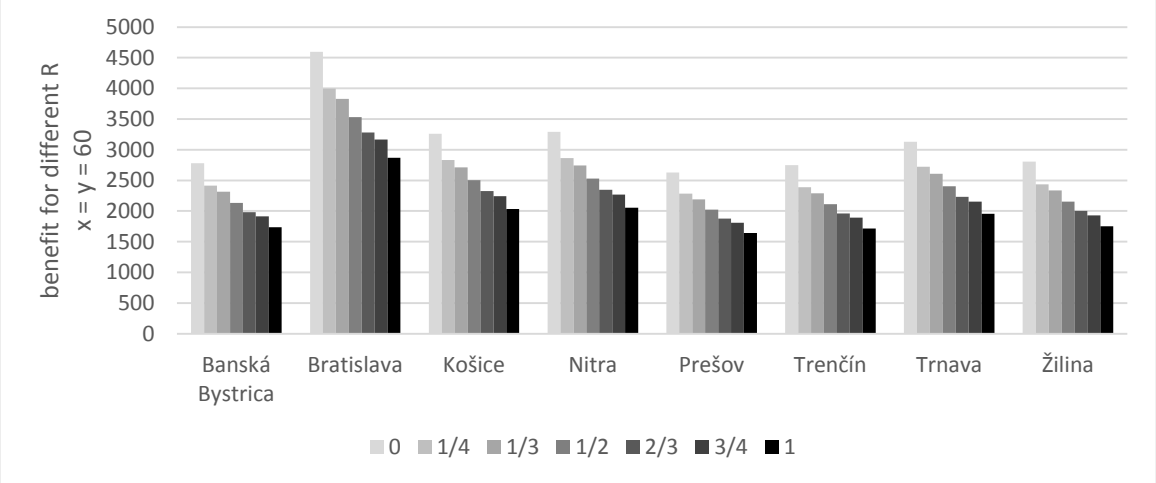


Source: own research.

The value of benefits depends strictly on a city. The highest benefits are in Bratislava and the smallest are in Prešov. We can see that a sixty-year-old marriage can receive a higher benefit if they have sixty square meter apartment in Bratislava than if they have a ninety square meter apartment in other cities. In Bratislava the differences between real and average price are not significant. In other cities, the benefit calculated under the assumption of real price is higher for small flats than it is calculated using average price. For sixty square meter property the benefit calculated using real price is higher only in Banská Bystrica and Bratislava. In other cities it is the opposite. The benefits is a little bit smaller in case of medium flats.

The value of benefits decreases with the rise of R. It can be observed in Figure 3. The benefits are calculated for sixty square meter flats by the use of average price.

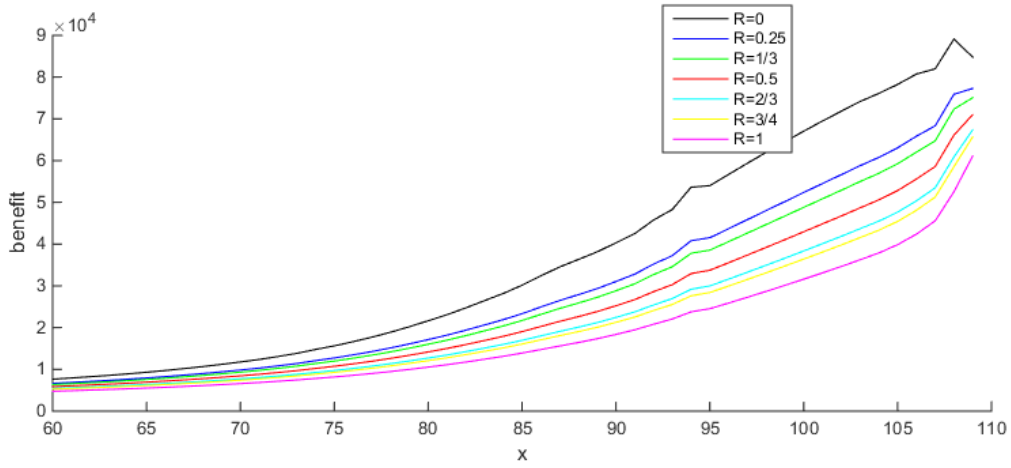
Figure 3. The benefit of marriage reverse annuity contract for different R



Source: own research.

In Figure 4 the benefit is presented for all marriages when wife and husband are the same age $x = y \in \{60, 61, ..., 110\}$. The benefit was calculated for Bratislava.

Figure 4. The benefit of marriage reverse annuity contract for different R and $x = y$



Source: own research.

When R is equal to zero, then the benefit grows faster for elderly people. When spouses are over 105 years, we receive strange results because of inaccuracies in the life tables. However, a product like marriage reverse annuity contract has no sense for very old people. The probability that both spouses are alive is very low. Hence, companies do not want to sell this contract. Moreover, people do not need such money.

Usually, spouses are at different age. Therefore we calculate the benefits for $x \in \{65, 70, 75, 80, 85\}$ and $y \in \{65, 70, 75, 80, 85\}$. The calculations are made by the use of unit annuity contract $\alpha = 50\%, W = 1$ and size of apartment equals 1 square meter for $R = 1$. Then the benefits are calculated in the case of Banská Bystrica, Bratislava and Košice, when the size of apartment is equal to 100 m^2. We assume the average prices from last column of Table 1. The results are presented in Table 2.

Table 2: The benefit of marriage reverse annuity contract for $R = 1$

		woman's age x				
	unit benefit	65	70	75	80	85
men's age y	65	0,05014	0,05547	0,06039	0,06428	0,06683
	70	0,05263	0,05988	0,0674	0,07403	0,07876
	75	0,05448	0,06359	0,07424	0,08498	0,0937
	80	0,05568	0,06625	0,0799	0,09562	0,11038
	85	0,05638	0,06789	0,08383	0,10423	0,12611
		benefit in Banska Bystrica				
men's age y	65	3344	3700	4028	4287	4457
	70	3510	3994	4496	4938	5253
	75	3633	4241	4952	5668	6250
	80	3714	4419	5329	6378	7362
	85	3760	4528	5591	6952	8412
		benefit in Bratislava				
men's age y	65	5533	6121	6664	7093	7374
	70	5807	6607	7438	8169	8691
	75	6011	7017	8192	9378	10340
	80	6144	7310	8817	10552	12180
	85	6221	7492	9250	11502	13916
		benefit in Košice				
men's age y	65	3923	4340	4726	5030	5229
	70	4118	4685	5274	5793	6163
	75	4263	4976	5809	6650	7332
	80	4357	5184	6252	7482	8637
	85	4411	5313	6559	8156	9868

Source: own research.

The highest benefit is in Bratislava, due to the most expensive price per square meter apartment. Adequately, the smallest benefit is in Banska Bystrica from those presented in

Table 2). The woman's age has a higher impact on the benefit, for example the benefit is higher for $y = 65$ and $x = 85$ than contrary. If the woman is older, the benefit is higher (in the case of $R = 1$).

Let us compare the results obtained for ten-year marriage reverse mortgage and whole life marriage reverse annuity contract for the Joint-Life Status $(R = 1)$ and when spouses are the same age. The results are presented in Table 2. The benefits of both contracts for the Last Surviving Status $(R = 0)$ in Banská Bystrica are presented in Table 3.

Table 3: The benefits of marriage reverse mortgage (r. mortgage) and marriage reverse annuity contract (r. annuity c.) for $R = 1$

cities	Banská Bystrica	Bratislava	Košice	Nitra	Prešov	Trenčín	Trnava	Žilina
				***x* = *y* = 60**				
r. mortgage	7138	11810	8374	8456	6758	7063	8043	7202
r. annuity c.	4631	7661	5433	5489	4385	4583	5218	4673
				***x* = *y* = 65**				
r. mortgage	7622	12610	8942	9033	7216	7542	8587	7690
r. annuity c.	5633	9320	6609	6677	5334	5574	6347	5684
				***x* = *y* = 70**				
r. mortgage	8502	14067	9975	10077	8050	8413	9580	8579
r. annuity c.	7124	11785	8357	8443	6744	7049	8026	7188
				***x* = *y* = 75**				
r. mortgage	10229	16923	12000	12123	9685	10122	11525	10321
r. annuity c.	9452	15638	11089	11202	8949	9353	10650	9537
				***x* = *y* = 80**				
r. mortgage	13412	22189	15735	15895	12698	13271	15111	13533
r. annuity c.	13086	21649	15352	15509	12389	12948	14744	13203

Source: own research.

Table 4: The benefits of marriage reverse mortgage (r. mortgage) and marriage reverse annuity contract (r. annuity c.) for $R = 0$ in Banská Bystrica

$R = 0$	r. mortgage	r. annuity c.	Relative increase
$x = y = 60$	6208	2888	114.91%
$x = y = 65$	6254	3344	87.01%
$x = y = 70$	6377	3994	59.66%
$x = y = 75$	6714	4952	35.58%
$x = y = 80$	7516	6378	17.84%

Source: own research.

The relative increase between both benefits is determined as follows

$$\text{relative increase} = \frac{\text{benefit of reverse mortgage - benefit of reverse annuity contract}}{\text{benefit of reverse annuity contract}}. \qquad (14)$$

The relative increase is almost the same in all city, therefore in Table 3 it is presented only for Banská Bystrica. The benefit of marriage reverse mortgage is higher than the benefit of marriage reverse annuity contract. The differences between benefits are substantial for younger spouses. This is due to the shorter time of receiving it. The elderly people have shorter life expectancy, therefore the differences in benefits of both contracts are smaller. The relative increase decreases with rise of spouses' age, however they are more than twice lower in the case of the Joint-Life Status than in case of the Last Surviving status (for younger marriages). In case of the Joint Life Status the relative increase between benefits equals 54.14%, while in the case of the Last Surviving Status it is 114.91% for $x = y = 60$. For $x = y = 80$ the relative increase between benefits for $R = 1$ is only 2.49% (cf. Table 2) and for $R = 0$ it is still high and equals 17.84% (cf. Table 3) - it is over seven times more.

Let us compare the benefits for both status on an example of marriage reverse mortgage in Banská Bystrica. The benefits are as follows:

$$\ddot{b}_{(60,60):\overline{10}|} = 7138, \ddot{b}_{(60,60):\overline{10}|} = 13412, \text{ when } R = 0,$$

$$\ddot{b}_{(60,60):\overline{10}|} = 6208, \ddot{b}_{(60,60):\overline{10}|} = 7516, \text{ when } R = 1.$$

The Last Surviving Status allows receiving the benefit longer but it is smaller than in case of the Joint-Life Status. It increases more slowly with the rise of spouses' age. Hence the differences between benefits are considerable for elderly people.

Acknowledgements

The paper was financially supported by the grant scheme VEGA 1/0859/16 "Dynamics of nonlinear economic processes" of the Ministry of Education, Science, Research and Sport of the Slovak Republic.

6. Conclusion

In the paper the model of reversionary annuity has been applied to calculate the benefits of two marriage contracts, i.e. reverse annuity contract and reverse mortgage. The amount of annuity depends very much on the spouses' place of residence. It is related to different prices of a square meter of an apartment, which is shown in numerical calculations.

All calculations were made for the real Slovak data from 2014 and 2017 by the use of own interfaces written in MATLAB. The benefits depend on the fraction R (as the fraction R increases, the amount of annuity decreases), and they are higher for elderly people. They increase more slowly with the rise of spouses' age in case of higher R. The benefits received from reverse mortgage are considerably higher than those followed from reverse annuity contract.

The calculation of benefits is made for married couples under the assumption that their future lifetimes are independent random variables. However, the Svensson model can also be applied in the case when future lifetime of spouses is dependent.

References

[1] Anderson N., Breedon F., Deacon M., Derry A., Murphy G. 1996. Estimating and interpreting the yield curve. Chichester: John Wiley & Sons.

[2] Blake, D., Cairns A., Coughlan A, Dowd K., MacMinn R. 2013. The New Life Market. The Journal of Risk and Insurance, vol. 80, iss. 3, pp. 501-557.

[3] Bowers, N. L., Gerber, H. U., Hickman, J. C., Jones, D. A., Nesbit, C. J. 1986. Actuarial Mathematics. Itasca, Illinois: The Society of Actuaries. ISBN 978-09-3895-946-5.

[4] Charupat N., Kamstra M.J., Milevsky M.A. 2016. The Sluggish and Asymmetric Reaction of Life Annuity Prices to Changes in Interest Rate. The Journal of Risk and Insurance, vol. 83 iss. 3, pp. 519-555.

[5] De Rezende, R. B., Ferreira, M. S. 2013. Modeling and Forecasting the Yield Curve by an Extended Nelson-Siegel Class of Models: a quantile autoregression approach. Journal of Forecasting, 2013, vol. 32, iss. 2, pp. 111-123.

[6] European Central Bank 2018. [cit. 5.02.2018] https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html.

[7] Hannewald K., Post T., Sherris M. 2016. Portfolio Choice in Retirement – What is the Optimal Home Equity Release Product?. The Journal of Risk and Insurance, vol. 83, iss. 2, pp. 421-446.

[8] Human Mortality Database [cit. 5.02.2018], http://mortality.org.

[9] Luciano, E., Spreeuw, J., Vigna E. 2016. Spouses' Dependence across Generations and Pricing Impact on Reversionary Annuities. Risks, vol. 4 (2), pp. 1-18.

[10] Marciniuk, A. 2017. Marriage Reverse Annuity Contract and Reverse Mortgage – Application of a Generalized Model of Reversionary Annuity. In: Gardoń, A., Kozyra, C., Mazurek, E. (eds.) Applications of Mathematics and Statistics in Economics 2017. Conference Proceedings. Wrocław: Wrocław University of Economis Press. ISBN 978-83-7695-693-0, pp. 297-306, DOI:10.15611/amse.2017.20.24.

[11] Realitysk 2018. [cit. 5.02.2018] https://reality.sk.