PERIODIC AND THRESHOLD REBALANCING BASED ON POLAR INVESTMENT STYLES

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Abstract

The paper investigates usefulness of a rebalancing strategy that conjoins polar investment styles in both asset pre-selection and portfolio selection. It is customary in investing to adhere to a particular investment style defined on the basis of a measurable quantifiable characteristic (using screening) and to invest into assets that have this characteristic high or low, respectively. The strategy assumes that the portfolio is created by investing into both such polar classes of assets in certain proportions and these proportions would be maintained by periodic and threshold rebalancing over the investment horizon. The usefulness of the rebalancing strategy is evaluated by a case study oriented on the US stock market.

Keywords*: investment style, screening, periodic rebalancing, threshold rebalancing, transaction costs, S&P 500 Index*

JEL Codes*: G11*

1. Introduction

At the risk of some simplification, every investment process can be broken down into four basal stages. Stage I is asset pre-selection and implies that candidate assets are identified in the universe of assets for subsequent portfolio selection. Asset pre-selection is frequently implemented by screening on the basis of a suitable fundamental or technical criterion, giving thus rise to some investment style. Stage II is portfolio selection, in which an available budget is allocated to single assets. Stage III is usually carried out by means of an optimization algorithm. This stage stretches over the investment horizon and rests in portfolio monitoring and revisions of its composition. Revisions are determined and implemented by dint of a rebalancing strategy to assure that the portfolio complies with the investor's preferences regarding expected return and risk. Being the final stage, Stage IV embodies portfolio performance evaluation and perhaps liquidation.

The paper develops a rebalancing strategy that inserts opposite investment styles into current rebalancing strategies. Presently, rebalancing is most often based upon investing to different categories of assets (such as equities, bonds or commodities). The proposed strategy does not centre upon different assets, but only upon equities that are further distinguished into disparate and opposite classes with the aid of a suitable screening criterion. Two criteria are employed to that end in order to define polar equity (investment) styles – viz., market capitalization and the P/B ratio. Whilst the former criterion classifies stocks as big caps and small caps, the latter criterion makes a distinction between growth and value stocks. Contemporary practice favours investing into assets of diverse categories with the hope of achieving a satisfactory level of diversification and then seeks to maintain some pre-determined proportions of shares between these categories (e.g. Tokat and Wicas, 2007). The

idea put forward in the paper is that investing into polar investing styles is also helpful in achieving desirable diversification and it even reduces transaction costs as trading with one category of assets is generally cheaper than dealing with several asset categories. Nonetheless, the idea is here restricted to equities only and exploits the stylized fact that big and small caps as well as growth and value stocks respond to market trends differently (Fabozzi, 1998, pp. 101-109). Their tendency to off-set one another is but the objective of diversification. A useful approach to practical investing implied by the outlined idea may be to invest into equities represented by polar investment styles in some proportions and to strive to maintain the pre-defined proportions over time using traditional methods of rebalancing.

Reliability of the proposed rebalancing approach is assessed in a case study oriented on the US stock market. The design is experimental and hinges upon tracking the S&P 500 Index using the traditional and most popular quadratic formulation. Several portfolios are fictively created by investing into S&P 500 constituents, rebalanced under different configurations and compared in terms of their performance. In order to ameliorate the subjectivity of choice and to gain more generalizable insights, the design implements and juxtaposes configurations varying by investment style, rebalancing strategy, portfolio size and sample period. As announced, two investment styles are taken into account and they are based on size (market capitalization) and pricing (the P/B ratio). This means that in portfolio construction big caps are blended with small caps and growth stocks are intermingled with value stocks. In defining new rebalancing strategies, the two investment styles are further conjoined with two approaches to rebalancing: periodic rebalancing (realized at a regular basis regardless of the actual market situation) and threshold rebalancing (initiated only if there is a deviation from the desired portfolio composition due to market development). Another factor is portfolio size that answers to the situation of a small investor, which entails considering only portfolios formed nominally out of 10, 20, 30 and 40 stocks. Finally, portfolios are constructed and evaluated for four different consecutive data samples represented by a two-year long in-sample period and a two-year investment horizon.

Using historical time series monthly frequency of data and considering the existence of transaction costs, the paper studies the performance of 1440 tracking portfolios and finds that periodic rebalancing boost return particularly of portfolios melding big and small caps and threshold rebalancing helps in volatility reduction.

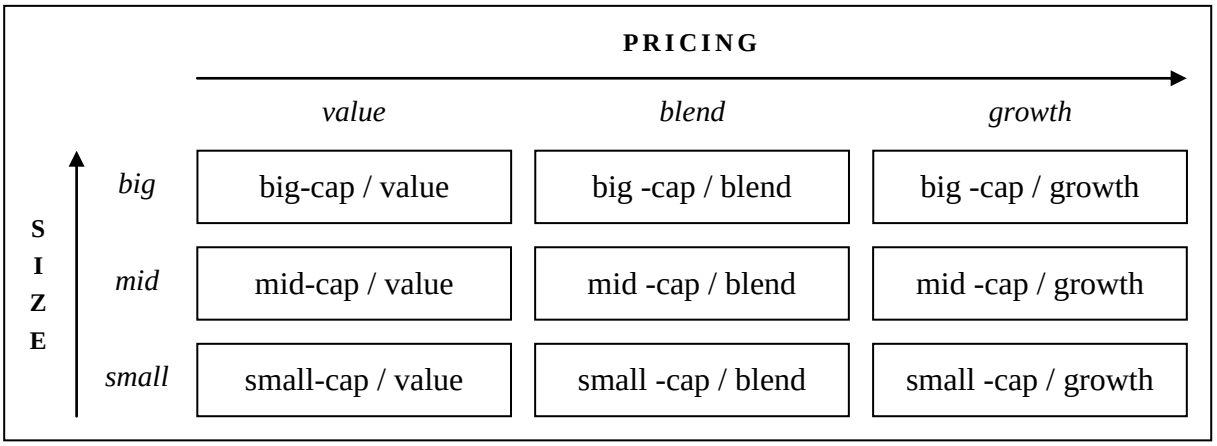
The remainder of the paper is organized into four more sections. Section 2 gives a brief explanation and substantiation of the two investment styles considered and is followed by Section 3 that clarifies the adopted methodological set-up. Section 4 describes the experimental design and submits the obtained results. Finally, Section 5 concludes.

2. Investment Styles and Rebalancing

An investment style is in fact a preference for a certain class of assets that is defined and reflected in a certain measurable criterion such as market capitalization or a suitable financial multiple. Perhaps needless to say, if the focus is limited to equities, it is frequently spoken of equity style in place of investment style. In practice several approaches may be contemplated at a time and combined as is also proven by the so-called "equity style box" devised by Morningstar Inc that is immensely popular with market practitioners (see TIAA-CREF, 2018, p. 9). The box takes form of a stock grid that classifies equities into nine possible combinations ranging from large-cap stocks (for the safest investments) to small-cap growth stocks (for the riskiest ones). Each combination represents a specific return-risk profile and suits the preferences of a different kind of investor. Figure 1 portrays the described equity box style. Plotted on the vertical axis, risk is associated with the size criterion, and big caps are considered less risky (stable) as opposed to small caps that are viewed more risky (volatile).

The horizontal axis captures the most fashionable classification of equities into value and growth stocks imposed by the relationship of market price to a fundamental. As fundamentals are usually considered book value of equity, net earnings or sales of the firm reported for a fiscal year, and they are related to market price through a multiple (a ratio indicator comparing the market price and fundamental). These comparisons lead to the price-to-book (P/B), price-to-earnings (P/E) and price-to-sales (P/S) ratios. In the present study, the classification is based upon the P/B ratio. A stock with a high value of the P/B ratio is tagged as a growth stock, and – vice versa – a stock with a low value of this multiple is labelled as a value stock. Investing into value stocks means purchasing stocks that are cheap relative to their fundaments and is based on the stylized fact that the market can be beaten by stocks that have prices low relative to their earnings, dividends, historical prices , book values (hence the P/B ratio) or other measures of their value.

Figure 1: Morningstar's equity style box



Source: TIAA-CREF (2018, p. 9)

The raison d'être of the nine-cell classification by size and pricing displayed in Figure 1 is to catch interactions of diverse investment styles so that a more favourable return-risk profile is attainable. Classical financial theory suggests that smaller firms are more risky than larger companies and this is the main point in explaining why small caps tend to earn better returns than big caps, although the actual explanations differ as is seen in Liu (2006) or Vassalou and Xing (2004). The role of investment styles recognized by the pricing criterion is also not negligible. In this respect, Hansen (1992) proved that 60 % differences in returns earned in a medium-term and long-term horizon may be explained through a diversity of investment styles. Sharpe (1992) goes even farther to claim that 90 % performance of equity funds is attributable to investment style and only 10 % comes from idiosyncratic characteristics of individual assets in the funds. In addition, empirical studies of the few last decades acknowledge that value-oriented strategies tend to generate higher returns than growth-oriented strategies. This evidence was amassed and confirmed in the research of Chan *et al*. (1991) or Lakonishok *et al*. (1994). All these studies demonstrated comparative performance of value equities over growth equities identified by dint of the P/B ratio for the Japanese and US markets. Notwithstanding, more recent studies contradict universality of these findings and suggest that there is some correlation with technical cycles and market trends, e.g. Chan and Lakonishok (2004).

The summarized observations underpin the ambition of the paper to make the best of two worlds represented by polar investment styles identified by market capitalization and pricing.

Small caps ("S") and big caps ("B") are identified by ordering stocks by their market capitalization and splitting them midway around the 50 % quantile. Growth stocks ("G") and value stocks ("V") are singled out by ordering stocks first by their P/B ratios and then dividing them by their accumulated market capitalization. This is the procedure advanced by Fabozzi (1998, p. 60).

Once the portfolio is optimized using some suitable allocation rule such as expected utility maximization or index tracking and created, the investor faces a decision whether he should keep the portfolio intact over the investment horizon or he should rebalance it whenever its return-risk profile is eroded. Rebalancing yields numerous benefits. First, in comparison to the buy-and-hold strategy it reduces volatility and has little or no unfavourable effect upon mean returns (see Dichtl *et al.*, 2013). Second, it decreases risk concentration and downside risk (see Bouchey *et al.*, 2012). The other side of rebalancing is that it induces transaction costs that off-set the input of rebalancing to preserving or improving performance over the investment horizon. Dichtl *et al.* (2013) divided rebalancing strategies into two chief groups: periodic and interval rebalancing. Periodic rebalancing means regular reallocation of assets with respect to pre-determined weights set at the very outset of the investment horizon. One approach to interval rebalancing is to set a non-trade region around target weights defined by means of a suitable threshold and to undertake a revision only if the portfolio deviates from these weights. For example, traditionally a certain proportion is invested into stocks and the rest into bonds. Say that these weights at the moment of portfolio creation are v and $1 - v$, respectively, and say that a threshold θ is introduced (such that $v \in [0,1]$ and $\theta \in (0,1)$ being relatively small). Over time as the market value of the portfolio develops, the proportions v and $1 - v$ change and the threshold θ controls the timing of an intervention. Usually at regularly spaced time intervals the portfolio is monitored and checked whether the actual proportions do not deviate from the intended proportions by more than $\pm\theta$. Whenever this is the case, the portfolio weights are optimized and reset to v and $1 - v$ for stock and bonds, respectively.

3. Methodology of Portfolio Selection and Rebalancing

It is assumed that two classes of equities are distinguished in a universe of choice and these classes represent polar categories dictated by size or the P/B ratio as exposited in the previous section. The proportions to be invested into the polar classes are v and $1 - v$ for big-cap / growth stocks or small-cap / value stocks. For simplicity, it is assumed that the same number m of equities is chosen in each category and that the weights in both classes of assets are determined separately. Denote the $(m \times 1)$ vector of weights for big-cap / growth stocks by $\boldsymbol{\omega}_1$, and, similarly, the vector of weights for small-cap / value stocks by $\boldsymbol{\omega}_2$. Both vectors are required to satisfy $\boldsymbol{\omega}_1'\mathbf{1} = 1$ and $\boldsymbol{\omega}_2'\mathbf{1} = 1$, wherein $\mathbf{1}$ is an $(m \times 1)$ vector of ones. The actual funds allocated to big-cap / growth stocks is then $v \cdot \boldsymbol{\omega}_1$ and into small-cap / value stocks $(1 - v) \cdot \boldsymbol{\omega}_2$. Throughout this process, the weights $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are optimized independently using the formulation of quadratic tracking for the m big-cap / growth stocks and the m small-cap / value stocks, respectively. The task is detailed e.g. in Boďa and Kanderová (2016, p. 48) and presupposes that a benchmark index is available. This function is taken over by the S&P 500 Index as is clarified in the next section.

Denote the moment of portfolio construction at the end of the in-sample period by subscript τ, denote the prices of individual assets at time τ by symbols $P_{\tau,1}, \ldots, P_{\tau,k}$ and the price of the benchmark as $P_{\tau,B}$. Because the portfolio is assembled of m big-cap / growth and m small-cap / value assets, in a non-trivial case $v \in (0,1)$ it must hold that $k = 2m$. In the trivial case $v \in \{0,1\}$, k is reset to $2m$ as well in order to enforce comparability. If the initial

investment is Ψ_τ, the following portfolio holdings are suggested: $h_{\tau,1} = \Psi_\tau \cdot \omega_1 / P_{\tau,1}$, ..., $h_{\tau,k} = \Psi_\tau \cdot \omega_k / P_{\tau,k}$. At the same time, a fictional investment into the benchmark is done and the holding $h_{\tau,B} = \Psi_\tau / P_{\tau,B}$ is made. In the previous formulas the notation ω_1, ..., ω_k symbolizes individual weights of the total k stocks. Of course, these weights respect the proportions v and $1 - v$. The symbol Ψ will also denote the value of the tracking portfolio at any time denoted carefully in the subscript. Adding "B" in the subscript after the time instance will indicate that the value of the benchmark investment is had in mind. Finally, assume that there is a percentage rate of transaction costs $\varphi \in [0,1)$ that applies to the value of investment changes. Symbols that were introduced for a particular time extend naturally in their validity also for some future times. In consistency with the previous outline, there are several possibilities how to maintain this portfolio by the investor until the end of the investment horizon. The first option is the buy-and-hold strategy and is sort of liberal, whereas the other two options consist in rebalancing and are concerned with whether the weights of polar classes of stocks deviate from the predetermined proportions v and $1 - v$, or not.

– The investor may choose not to revaluate the composition of the portfolio at all and opt for the buy-and-hold strategy. In such a case, transaction costs are incurred only at the moment of portfolio creation in the amount

$$\varphi \sum_{i=1}^{i=k} | h_{\tau,i} | \cdot P_{\tau,i} , \tag{1}$$

which reduces into $\varphi \cdot \Psi$ when there is a ban on short sales (or when all holdings are positive).

– Another possibility is to rebalance the portfolio at regular time intervals of length, say, $\Delta\tau$ ($\Delta\tau > 0$), no matter what the situation on the market is and how the tracking portfolio copies the index. In this case, at the next time $\tau + \Delta\tau$, the task of quadratic tracking is resolved with updated historical data separately for big-cap / growth and small-cap / value stocks, yielding new vectors of weights $\boldsymbol{\omega}_{\tau+\Delta\tau,1}$ and $\boldsymbol{\omega}_{\tau+\Delta\tau,2}$. This updating is done on a sliding basis, keeping the length of historical observations unaltered. New holdings $h_{\tau+\Delta\tau,1}$,..., $h_{\tau+\Delta\tau,k}$ are thus produced using the proportions v and $1 - v$, and the portfolio is revised accordingly. In addition to the initial transaction costs resulting from the first portfolio construction given by (2), at the moment of revision, $\tau + \Delta\tau$, rebalancing transaction costs arise in the amount

$$\varphi \sum_{i=1}^{i=k} | h_{\tau+\Delta\tau,i} - h_{\tau,i} | \cdot P_{\tau+\Delta\tau,i} . \tag{2}$$

This, of course, goes on a sliding basis at rebalancing times $\tau + \Delta\tau$, $\tau + 2\Delta\tau$, ... until the end of the investment horizon.

– Finally, another possibility is to set a threshold and to monitor discrepancy between the value of the tracking portfolio and the value of the investment into the benchmark. For this, some maximum tolerance threshold θ (with $\theta > 0$) must be set. The monitoring is not performed on a continuous basis, but usually as with periodic rebalancing at regularly spaced instances $\tau + \Delta\tau$, $\tau + 2\Delta\tau$ for some $\Delta\tau > 0$. Whenever at such an instance the actual proportions of big-cap / growth and small-cap / value stocks deviate in absolute magnitude from the pre-determined values v and $1 - v$ by more than θ, it is an impetus for an intervention and the portfolio is rebalanced. With this intervention portfolio, additional transaction costs are associated in the same manner as explained about the formula (2).

There is one grave simplification with these strategies in comparison to their practical implementation since they should take into consideration also the fact that, at revision times, transaction costs must be paid and they should decrease the value of the portfolio. It is assumed here in the paper instead that there exists a separate account, from which these

transaction costs are covered. Only the final value of the tracking portfolio is confronted with the volume of transaction costs (in an inflation-free world), and the net value of the investment is computed by subtracting the transaction costs total from the portfolio value.

4. Empirical Configuration and Results

The study takes the S&P 500 as benchmark for a non-institutional investor who desires to form a small portfolio of S&P 500 constituents. Choosing the S&P 500 and its constituents is on account of its credit as this index is believed to represent well the patterns and sentiments of the US market. In order to control for the amount of transaction costs affecting ultimate performance, the investor is willing only to form a tracking portfolio of no more than 40 stocks. To this end, he uses 24 historical monthly logarithmic returns for a period of two years (the in-sample period) to identify the weights of portfolios that track the underlying S&P 500 quadratically. Using the procedure described in Section 2, the investor first classifies the universe of S&P 500 constituents into big and small caps as well as into growth and value stocks, and orders them notionally on the real axis. Following the outline of Section 3, he then sets up portfolios of different nominal sizes counting $5 + 5$, $10 + 10$, $15 + 15$ and $20 + 20$ stocks of polar categories and of varying proportions $\nu \in \{0, 0.25, 0.50, 0.75, 1\}$ of allocation into big-cap / growth stocks. All these portfolios are formed on the last day of the in-sample period that coincides with year-end. Afterwards, the portfolios are held and monitored in the course of the next two years (the out-of-sample period or investment horizon) and possibly held without change or rebalanced in conformity with the strategy applied.

A total of four samples are employed and are referred to as “periods”. These samples span a period of four years, with the first two years representing the in-sample period of 24 monthly returns for portfolio selection and the last two years standing for the out-of-sample horizon of active investing and rebalancing. The samples start at 2011 (the start of the first in-sample period) and end at 2017 (the end of the last out-of-sample period), which is the reason they are denoted as "20112014" to "20142017". At the end of each in-sample period, the S&P 500 Index was screened for its constituents and the effective number of constituents was somewhat smaller than 500 (owing to unavailability of data or changes in the index over time). In consequence, the effective universe of S&P 500 constituents ranged from 450 (with period “20122015”) to 458 (with periods “20112014” and “20142017”).

For each sample at the end of the in-sample period mixing portfolios are identified and created either using a combination of big and small caps ("BS" mixing portfolios) or growth and value stocks ("GV" mixing portfolios) using as many as five mixing proportions and four nominal sizes. These portfolios are optimized under a ban on short sales and made by investing the initial budget of US $ 10,000 with the rate of transaction costs set to $\varphi = 0.4$ %. These portfolios are then held intact over the out-of-sample period, which amounts to applying the buy-and-hold strategy, or rebalanced. Four periodic rebalancing strategies are considered differentiated by whether revisions are undertaken periodically every month (1M), every quarter (3M), every half-year (6M) and every year (12M/1Y). Moreover, these periodic revisions are complemented by threshold rebalancing strategies with four different choices for the threshold, viz. $\theta = 0.005$, $\theta = 0.01$, $\theta = 0.015$, and $\theta = 0.02$. Note that when a portfolio is rebalanced under a certain value for threshold θ, it is also rebalanced for any smaller threshold. In total, the experiment resulted in a total of $4 \times 5 \times 4 \times (1 + 4 + 4) = 720$ "BS" mixing portfolios and in the same number of "GV" mixing portfolios. The multiples here are sample – mixing proportion – nominal size – rebalancing strategy, respectively.

It is worthwhile noting that at the end of the out-of-sample period no portfolio was actually liquidated and so transaction costs were spared. Only the terminal value of the portfolio was

ascertained and adjusted by the total amount of transaction costs. Three measures of portfolio performance were allowed for in the assessment: mean return, volatility and net cumulative return, all being expressed in annualized form (as p.a.). Net cumulative return differs from mean return by being calculated as a compound annual growth rate inclusive of transaction costs.

In computations and preparing graphical presentations, the software R version 3.0.1 (R Core Team, 2013) was employed with several of its libraries, `quantmod`, `quadprog`, `timeSeries` and `PerformanceAnalytics` .

The experiment led to an exhaustive computational output, most of which is not reported for its substantial size. Before exhibiting and commenting selected results in Figures 2 and 3 as well as in Table 1, the next few summarizations of results will relate to 160 portfolios that were formed initially at the end of any in-sample period regardless of the method of their management over the investment horizon. Since there were two polar investment styles, four portfolio nominal sizes and five mixing proportions applied to four different periods, the number of portfolios ensues from the calculation $2 \times 4 \times 5 \times 4 = 160$.

Out of these 160 portfolios created over the entire period under evaluation, better results with respect to the criterion of mean return were achieved by any method of rebalancing in 132 cases (82.5 %), amongst which it were "GV" mixing portfolios that dominated (86 %). In terms of frequency, the best rebalancing strategy was annual periodic rebalancing and this regularity manifested itself mostly with "BS" mixing portfolios. Also "GV" mixing portfolios were rebalanced more successfully in terms of mean return when rebalanced periodically, but at different frequencies.

In terms of volatility, more satisfactory results were achieved by rebalancing in the case of 127 portfolios out of the total 160 portfolios constructed (79 %). It were threshold rebalancing strategies with various threshold levels that dominated in volatility reduction.

When net cumulative value was taken as a measure of performance, better results obtained by rebalancing were detected for 123 portfolios out of 160 (77 %). Here a higher number of non-rebalanced portfolios (those managed by the buy-and-hold strategy) was owing to transaction costs. Also under criterion annual periodic rebalancing strategy was found for annual periodic rebalancing. The effect of rebalancing in the sense of a higher net cumulative value when compared to the buy-and-hold strategy evinced itself more remarkably with "GV" mixing portfolios. As many as 81 % rebalanced "GV" mixing portfolios yielded a more favourable net value than the buy-and-hold strategy, whereas for "BS" mixing portfolios this percentage was only 73 %.

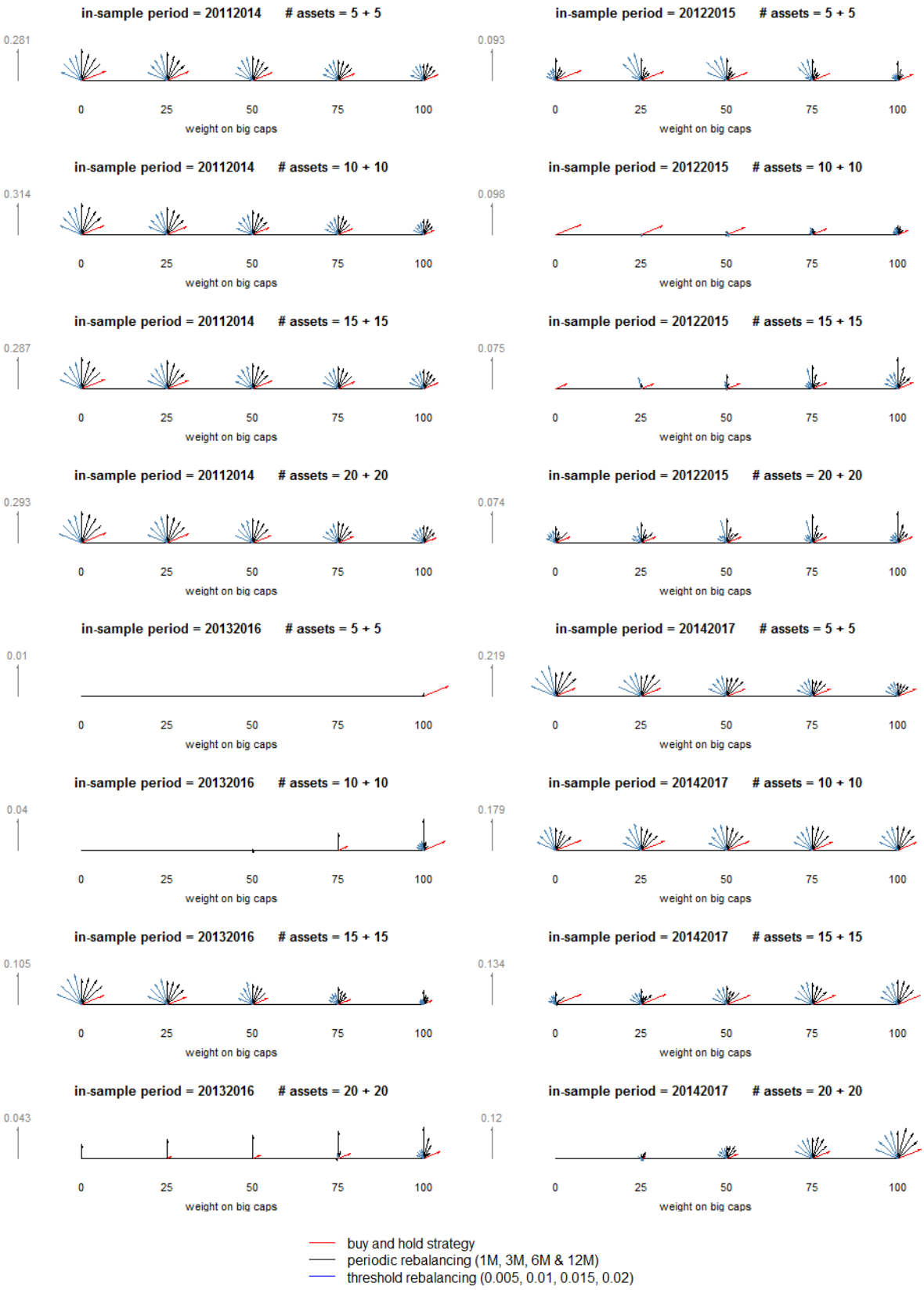
It is also found that the suitability of a rebalancing strategy was gravely affected by the period in which portfolios are formed and held. In period "20112014" rebalancing improved the mean return, volatility and net cumulative return of 90 %, 90 % and 75 % "BS" mixing portfolios, respectively. In contrast, the performance of "GV" mixing portfolios was not so much improved by rebalancing, even mostly deteriorated as was the case with volatility. Some form of rebalancing decreased volatility in comparison to the buy-and-hold strategy only for 30 % "GV" mixing portfolios. Period "20132016" showed patterns very similar to period "20112014", but in periods "20122015" and "20142017" rebalancing was detected contributive mostly to "GV" mixing portfolios. Rebalancing improved the performance of between 100 % and 90 % "GV" mixing portfolios irrespective of the criterion of performance. To exacerbate the contrasting behaviour, rebalancing bettered the performance of less than 50 % "BS" portfolios for all the three criteria.

For brevity and clarity of demonstration, only partial results are presented by combining a graphical format in Figures 2 and 3 and a tabular format in Table 1. Figures 2 and 3 are drawn for the annualized mean returns of portfolios and display arrowed star plots for the

annual mean returns of the "BS" and "GV" mixing portfolios, respectively. These star plots are produced for each sample, portfolio nominal size and mixing proportion, and visualize annual mean returns as arrows starting from the same origin. These arrows are grouped for each sample and portfolio nominal size and normalized so that the highest annual mean return for every such grouping and horizontal bar has an arrow equally long. The particular value to which other mean returns on a horizontal bar normalized is displayed on the left-hand side above the vertical arrow that acts as a norm. Negative mean returns are simply not plotted. Colours of the arrows comply with different rebalancing strategies, but periodic rebalancing and threshold strategies are not further separated. There are clear patterns that are discernible as an apparent effect of the period and mixing proportions upon the performance of created portfolios. First of all, negative mean returns were in point of fact a matter of "BS" mixing portfolios and plagued especially periods "20122015" and "20132016". In other cases (other two periods for portfolios conflating big and small caps, and all four periods for portfolios blending growth and value stocks) negative mean returns were scarce. Second, there is evidence of a fairly monotonous response of mean returns to adjustments in mixing proportions. The best values of mean return are frequently found at unipolar investments when all is invested into small caps (e.g. periods "20112014" and "20132016" for "BS" mixing portfolios) or into growth stocks (virtually all periods for "GV" mixing portfolios). What escapes the plotting capacity of Figures 2 and 3 is the fact that such portfolios have the highest volatility. It is not surprising that the lowest levels of volatility are detected for mixed portfolios, which is on account of their better diversification.

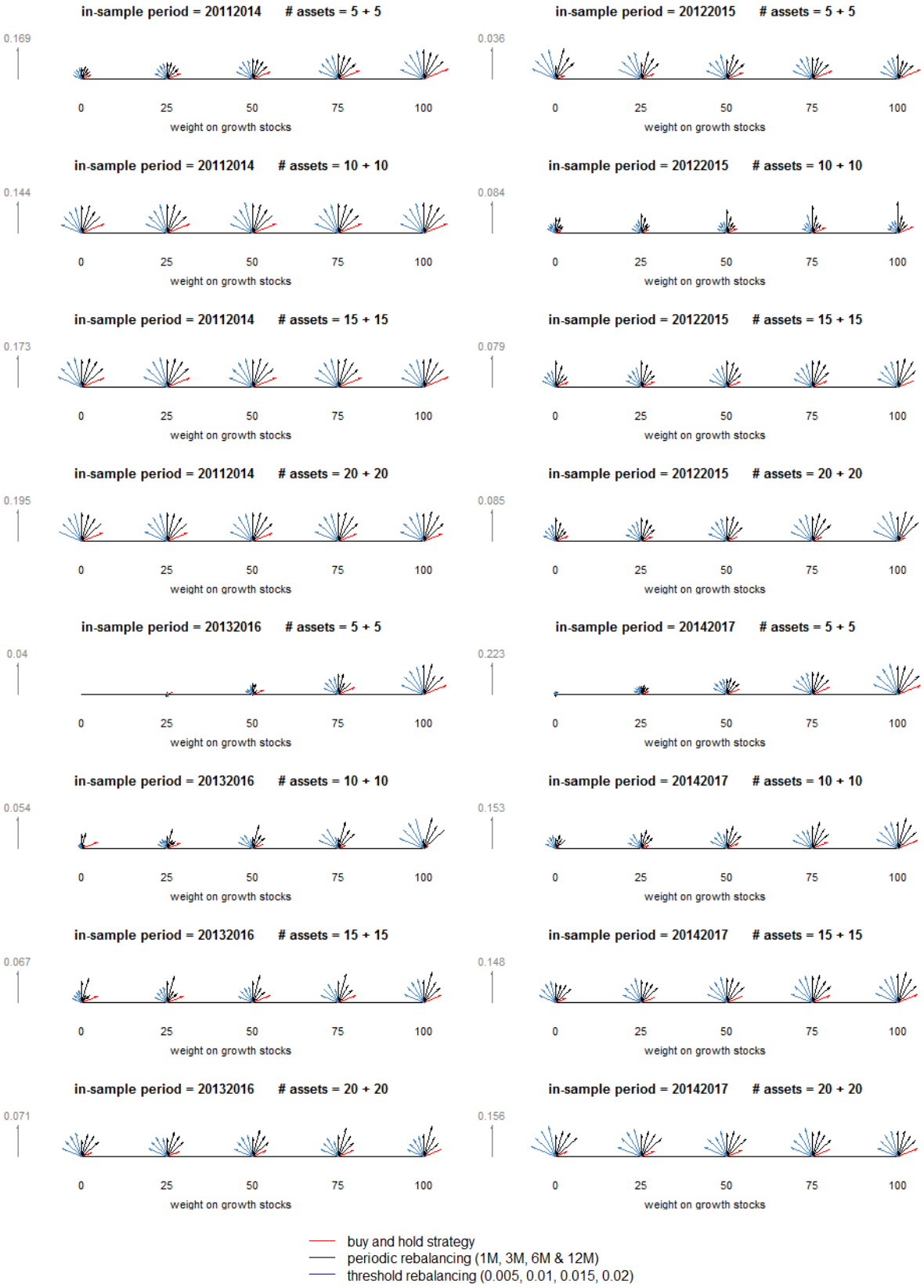
Table 1 reports for both polar investment styles, individual periods, portfolio nominal sizes and mixing proportions the strategies that turned out to be most favourable for the three measures of performance under consideration. Also this report is confirmative of the effect of investment style and period upon portfolio performance. Whereas for the "BS" investment style in periods "20112014" and "20132016" improving (both mean and net cumulative) return of a portfolio meant mostly implementing a periodic rebalancing strategy, but decreasing volatility required usually threshold rebalancing or no rebalancing at all, for the "GV" investment style in these two periods such a picture did not arise. It seems that there was no favourable rebalancing strategy that would have boosted return safely and uniformly. For mean and net cumulative returns periodic rebalancing or no rebalancing appeared more appealing and reliable. In contrast to the previous case of "BS" mixing portfolios, in periods "20112014" and "20132016" now the most apparent decreases of volatility were thanks to threshold rebalancing. As far as the other two periods "20122015" and "20142016" are considered, for the "BS" investment style the buy-and-hold strategy may have been considered uniformly preferable, whilst for the "GV" investment style it was mostly a periodic rebalancing strategy (in terms of mean and net cumulative return) or a threshold rebalancing strategy (through the optic of volatility).

Figure 2: Annual mean returns for "BS" mixing portfolios (negative values not plotted)



Source: The authors

Figure 3: Annual mean returns for "GV" mixing portfolios (negative values not plotted)



Source: The authors

Table 1: Most successful strategies for individual configurations the experiment

Period	# assets	"BS" mixing portfolios for mixing proportions on big caps 0 - 0.25 - 0.50 - 0.75 - 1.00		
		Mean return (% p.a.)	**Volatility (% p.a.)**	**Net cumulative return (% p.a.)**
"20112014"	5 + 5	P1Y-P1Y-T-BH-BH	T-T0.02-BH-BH-BH	P1Y-P1Y-T-BH-BH
	10 + 10	P1Y-P1Y-P1Y-P1Y-P6M	P1Y-P1Y-P1Y-P1Y-P1Y	P1Y-P1Y-P1Y-P1Y-P6M
	15 + 15	P1Y-P1Y-P1Y-P1Y-P1Y	T-T-T0.02-P1Y-BH	P1Y-P1Y-P1Y-P1Y-P1Y
	20 + 20	P1Y-P1Y-P1Y-P1Y-P6M	T-T0.02-T0.02-BH-BH	P1Y-P1Y-P1Y-P1Y-P6M
"20122015"	5 + 5	BH-BH-T0.015-BH-P1Y	BH-BH-BH-T0.02-T	BH-BH-T0.015-BH-P1Y
	10 + 10	BH-BH-BH-BH-BH	P3M-T0.005-T0.01-T0.005-P1M	BH-BH-BH-BH-BH
	15 + 15	BH-BH-BH-P1Y-P1Y	P1Y-T0.005-BH-T0.005-T	BH-BH-BH-P1Y-P1Y
	20 + 20	BH-P1Y-P1Y-P1Y-P1Y	BH-BH-BH-BH-BH	BH-P1Y-P1Y-P1Y-P1Y
"20132016"	5 + 5	T-P1M-P1M-BH-BH	T-T0.02-T0.02-T0.015-P3M	T-P1M-P1M-BH-BH
	10 + 10	P1Y-P1Y-P1Y-P1Y-P1Y	T-T0.02-BH-T0.005-P1M	P1Y-P1Y-P1Y-P1Y-P1Y
	15 + 15	T-T0.005-T0.005-P1Y-P1M	T-T-T0.02-P6M-P6M	T-T0.005-T0.005-P1Y-P1M
	20 + 20	P1Y-P1Y-P1Y-P1Y-P1Y	P1Y-P1Y-P1Y-P1Y-T	P1Y-P1Y-P1Y-P1Y-P1Y
"20142017"	5 + 5	T-T0.02-P1M-BH-BH	T-T0.01-BH-T0.02-P1Y	T-T0.02-BH-BH-BH
	10 + 10	BH-BH-BH-BH-P3M	BH-T0.015-T0.015-T0.015-BH	BH-BH-BH-BH-P1Y
	15 + 15	BH-BH-BH-BH-BH	BH-BH-BH-BH-BH	BH-BH-BH-BH-BH
	20 + 20	P3M-P3M-P3M-P3M-P6M	T-P1M-T0.01-P1M-BH	P3M-P3M-BH-BH-P6M
Period	**# assets**	**"GV" mixing portfolios for mixing proportions on growth stocks 0 - 0.25 - 0.50 - 0.75 - 1.00**		
		Mean return (% p.a.)	**Volatility (% p.a.)**	**Net cumulative return (% p.a.)**
"20112014"	5 + 5	P6M-BH-BH-BH-BH	T-BH-BH-BH-BH	P6M-BH-BH-BH-BH
	10 + 10	P6M-T0.005-T0.005-P3M-P3M	BH-BH-BH-BH-BH	P6M-P6M-T0.005-BH-BH
	15 + 15	P1Y-P1Y-P3M-P3M-BH	BH-BH-BH-BH-BH	P3M-P3M-P3M-BH-BH
	20 + 20	T-T-P3M-P3M-P3M	P1M-P1M-T0.005-P1Y-P1Y	P1Y-P1Y-P6M-P3M-P3M
"20122015"	5 + 5	P6M-T-T0.02-BH-BH	P3M-P3M-T0.005-T0.005-T	P6M-P6M-P6M-BH-BH
	10 + 10	P6M-P1Y-P1Y-P1Y-P1Y	BH-T-T-T-T	P6M-P1Y-P1Y-P1Y-P1Y
	15 + 15	P1Y-P1Y-P1Y-P6M-P3M	T-T-T-T-T	P1Y-P1Y-P1Y-P6M-P3M
	20 + 20	P1Y-P1Y-P6M-P6M-P3M	BH-T-T-T-T	P1Y-P1Y-P6M-P6M-P6M
"20132016"	5 + 5	BH-BH-BH-P6M-P6M	T-BH-BH-BH-T	BH-BH-BH-BH-P6M
	10 + 10	BH-P6M-P6M-P6M-P6M	T-T0.005-T0.01-T0.005-T	BH-P6M-P6M-P6M-P6M
	15 + 15	P6M-P6M-P6M-P6M-P6M	T-T0.005-T0.01-T0.005-P3M	P6M-P6M-P6M-P6M-P6M
	20 + 20	T-P6M-P6M-P6M-P6M	T-T0.005-T0.01-T0.005-P3M	P3M-P6M-P6M-P6M-P6M
"20142017"	5 + 5	P1M-P1M-P1M-T0.005-P1M	T-T0.02-P6M-BH-BH	P1M-T0.015-T0.015-T0.015-P1M
	10 + 10	P6M-P6M-P6M-P6M-P6M	T-T0.02-T0.01-P6M-P6M	P6M-P6M-P6M-P6M-P6M
	15 + 15	P1M-T0.005-T0.005-T0.02-P6M	T-T0.02-T0.015-P6M-P6M	P1M-T0.005-T0.005-P6M-P6M
	20 + 20	T-P1M-T0.015-T0.005-P6M	BH-BH-BH-P3M-P3M	P1M-T0.015-T0.02-T0.005-P6M

Note: The meaning of abbreviations is as follows: "BH" stands for the buy-and-hold strategy, "P1M"/"P3M"/"P6M"/"P1Y" stand for periodic rebalancing undertaken regularly each month, quarter, half-year and year, respectively, and "T0.005"/ "T0.01"/"T0.015"/"T0.02" represent threshold rebalancing at different threshold levels. In addition, "T" indicates that all threshold rebalancing strategies were equally successful.

Source: The authors

In addition to the previous analysis, a search was initiated in a regression context to explain the factors of performance of mixing portfolios and rebalancing strategies. Two traditional methods of regression model selections were put to use and encompassed LASSO and optimization of the Bayesian information criterion (BIC). The search was conducted separately for 720 "BS" and 720 "GV" mixing portfolios with respect to mean return, volatility and net cumulative return. All these return-risk attributes were therein denominated in annual percentages. The fitted regression models identified by either selection criterion are reported in Table 2. As follows from Table 2, the explanatory factors are the mixing proportion (i.e. weight), nominal portfolio size (i.e. # assets), approach to rebalancing and period. With the exception of nominal portfolio size, all the explanatory factors are represented through dummy variables, which is the reason why these models miss the intercept. The quality of fit of each model can be assessed prima facie by the reported adjusted coefficient of determination (R-squared).

What is obvious is the effect of period upon the performance of created portfolios, which somewhat varies between the "BS" and "GV" investment styles as well as the three criteria of

performance. In periods "20112014", "20122015" and "20142017" both return characteristics and volatility tended to be higher for either investment style. This is indicative that earning higher returns was indispensably linked with accepting higher risk. The coefficients of the fitted regression models are also affirmative of the findings formulated afore. First of all, with the volatility of "GV" mixing portfolios was smaller for non-extreme mixing proportions (with a 25 %, 50 % or 75 % share invested into growth or value stocks), but their mean and net cumulative return could be best increased by investing a full share of 100 % into growth stocks. This regularity is not discovered for "BS" mixing portfolios, but it is apparent that investing all funds into big cap stocks merely is at the expense of higher volatility without any help to return. In addition, the buy-and-hold strategy seems to advantage "BS" mixing portfolios in terms of mean and net cumulative return, but to impair "GV" mixing portfolios. Finally, periodic rebalancing strategies appear to increase both volatility and net cumulative return for both "BS" and "GV" mixing portfolios.

Table 2: Explanatory factors of portfolio performance identified in a regression context

Variable	"BS" mixing portfolios						"GV" mixing portfolios					
	Mean return (% p.a.)		Volatility (% p.a.)		Net cumulative return (% p.a.)		Mean return (% p.a.)		Volatility (% p.a.)		Net cumulative return (% p.a.)	
	LASSO	BIC	LASSO	BIC	LASSO	BIC	LASSO	BIC	LASSO	BIC	LASSO	BIC
Weight 0 on B/G stocks			6.63	6.50	0.23		-0.71		1.07	13.28	-1.00	-1.80
Weight 0.25 on B/G stocks			4.28	4.15	0.37			0.70	0.42	12.63		-0.80
Weight 0.50 on B/G stocks			2.03	1.90	0.31		0.76	1.61		12.20	0.60	
Weight 0.75 on B/G stocks			0.41				1.69	2.54	0.05	12.27	1.57	1.13
Weight 1 on B/G stocks					-0.47		2.59	3.44	0.59	12.80	2.51	2.08
# assets			-0.09	-0.25	-0.02		0.28	0.26	-0.03	-0.05	0.24	0.27
Buy-and-hold strategy	0.64	1.88			2.68	2.74	-0.77	-1.26	0.16		-0.22	
Periodically rebalanced 1M			0.07						0.18			
Periodically rebalanced 3M									0.09			
Periodically rebalanced 6M					0.31		0.41		0.11		0.74	1.03
Periodically rebalanced 1Y			0.00		1.78	1.85			0.27		0.07	
Threshold rebalanced 0.005					0.55							
Threshold rebalanced 0.01												
Threshold rebalanced 0.015							0.02					
Threshold rebalanced 0.02							0.00					
Period "20112014"	19.84	20.80	9.74	12.13	21.16	21.24	10.16	9.85	8.86	-3.04	10.38	10.69
Period "20122015"	1.55	2.51	11.11	13.51	1.31	1.40	0.12		11.96			
Period "20132016"	-0.78	-2.02	14.68	17.07	-2.54	-2.75	-0.83	-1.49	11.84		-1.08	-1.10
Period "20142017"	9.21	10.17	11.23	13.62	9.39	9.47	6.93	6.61	8.09	-3.81	6.93	7.24
Adjusted R-squared	**0.84**	**0.84**	**0.96**	**0.96**	**0.85**	**0.85**	**0.95**	**0.95**	**0.99**	**0.99**	**0.94**	**0.94**

Note: Since the variables weight on B/G stocks, strategy (buy-and hold strategy, period rebalancing, threshold rebalancing) and period are all categorical variables, during their transformation to dummy variables the intercept was neglected to prevent the issue of perfect collinearity.

Source: The authors

5. Conclusion

The quintessential idea of the paper is to make the best of two worlds in combining contrasting poles of investment styles induced by a suitable screening criterion, in which the pre-selection stage (Stage I) of an investment process and the management stage (Stage III) are connected. No matter whether size or a multiple linking market price to a fundamental (here market capitalization and the P/B ratio) are employed in separating assets into classes, the effect is that there are opposing groups of assets with a differing potential of performance characteristics. By investing to such polar groups of assets in suitable mixing proportions the investor may afford himself an opportunity to create a portfolio that displays more satisfactory performance characteristics than portfolios whose composition is confined to one particular

class. It is obvious that – in order to take the full advantage of this mixing approach – the investment strategy must be coupled with a suitable management scheme resting in periodic or threshold rebalancing. This novel approach of periodic and threshold rebalancing based on polar investment styles is central to the paper and the desirability of the approach is demonstrated in a case study focusing upon the US stock market.

The case study employed the strategy of quadratic tracking of the S&P 500 Index using its constituent stocks and evaluated as many as 160 portfolios that were formed by mixing big-cap and small-cap S&P 500 constituents and growth and value S&P 500 constituents for four time periods, four portfolio nominal sizes and five mixing proportions. Three criteria of evaluation were chosen, i.e. mean return, volatility and net cumulative return. The results indicate that the rebalancing strategies considered were conducive to both increasing mean return and net cumulative return as well as to reducing volatility. That being said, success of individual strategies varied with period in conjunction with investment style, and it is therefore very difficult, if impossible, to make general statements. Nevertheless, it is still possible to identify certain tendencies. With respect to return annual periodic rebalancing prevails, especially in the case of portfolios arising as a mixture of big caps and small caps. In terms of volatility, higher desirability is discovered with threshold rebalancing strategies irrespective of the threshold level from the target weights that sets the timing for an intervention action. The case study utilized relatively small tolerance thresholds (ranging from 0.005 to 0.02), which resulted in more frequent rebalancing and incurred higher transaction costs. This is the very reason why this kind of a rebalancing strategy is least successful in regard to the criterion of net cumulative return.

Taking investment style into account, it is not possible to conclude firmly which of the investment styles considered is more recommendable or which needs rebalancing. The results emphasize that success of an investment strategy is inseparably associated with market trends and development. For instance, Zhang *et al*. (2008) find that a value-based investment style is more successful on bearish markets and a growth-based investment style asserts itself better on bullish markets. Tokat and Wicas (2007) claim that rebalancing strategies are more effective on trendless markets.

What must be said bluntly is that the strategies mixing polar investment styles as considered here are not always ideal and preferable over unipolar investment styles. The reason being, "BS" mixing portfolios produce in general comparatively the most desirable mean returns when the mixing proportion is set to 0 on big caps, which effectively means that small-cap portfolios are in fact best performers in terms of mean return. Likewise, "GV" mixing portfolios are most appealing under mean return mostly when the mixing proportion is chosen 1.00 on growth stocks, which reduces the composition of the portfolio to growth stocks only. The described unipolar pattern does not apply to volatility as volatility is found uniformly lower for mixed bipolar portfolios, which is a finding most reasonable as portfolios composed of small and big caps or of growth and value stocks are by right expectable to be more diversified. As noted in Section 2, to many the anticipated and sought-after effect of rebalancing is this volatility reduction. Finally, an interesting avenue for further research is to investigate the added effect of mixing best-performing unipolar classes identified by this study, i.e. small caps and growth stocks. The effect should be recognizable with both higher return and decreased volatility and hopefully, such portfolios could be superior.

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